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DRILL-BOOK IN ALGEBRA:

EXERCISES FOR

Class-Drill and Review.

ARRANGED ACCORDING TO SUBJECTS.

BY

MARSHALL LIVINGSTON PERRIN, A.M.

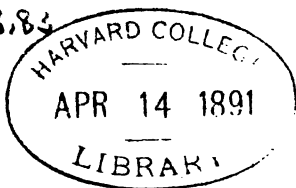


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PREFACE.

IN this country the French philosophical method of studying Mathematics is fast yielding to the demand for a practical, working knowledge. An evidence of this is the present style of examination papers for admission to colleges and schools.

Algebra is not an end in Mathematics, but a means only. Properly considered, Algebra is largely a multiplication-table to be used in other mathematical work. It is not sufficient, then, that the subject be understood: the processes must be readily handled. An unlimited variety of examples is needed to disengage the principle of the process from the individual example, and to ensure the proper application of the principle every time.

The advantage of having additional examples to be done in class or out of class, arranged subject by subject, will be recognized at once. To gather these from other algebras is a laborious task which this book is intended to supplant. With problems at hand, each pupil in class may be doing work different from his neighbor's, and new to himself, but with no more trouble to the teacher.

It is designed that each pupil be provided with a copy of the SCHOLAR'S EDITION *without answers*. It is desirable that a text-book should have answers; but in a supplementary work of this kind the pupil should be thrown upon his own resources, and the instructor be provided with the TEACHER'S EDITION *with answers*.

Too simple examples as well as instructions have not been given, since the intention is to supplement the text-book in use. At the same time there are no catch-questions, and

every example is to be solved in good faith. Variety of expression has been aimed at, to familiarize the student with different forms.

The exercises do not extend beyond elementary Algebra, but quite cover the ground required for admission to colleges and scientific schools in general. Nor does the collection claim to be exhaustive in all of the subjects: imaginary roots and quantities are omitted unless especially mentioned. Interesting phases of some subjects have been left out, keeping in mind the purpose of a practice book, efficient, yet not cumbersome. No space has been given to miscellaneous examples, which experience shows are not adapted to class-work in Algebra.

The collection is fully as much a compilation as it is original. German, French, and English works have been largely drawn from; but no problems have been taken from American authors. Indeed, in order not to defeat the object of this book, problems previously copied into American algebras have been as much as possible avoided.

The problems are arranged in the order of subjects usually adopted in text-books, and are divided into chapters, for convenience and as the subject of Algebra naturally divides itself. In many of these respects, it is believed, this collection differs from any work in use in this country, and it is hoped will form a useful drill and class-book.

M. L. PERRIN.

WELLESLEY HILLS, Mass., *June, 1883.*

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ALGEBRA PROBLEMS.

I.

NOTATION.

1. The sum of two numbers being 30, what will represent the second, if x represent the first?
2. Express the double of a .
3. By how much is x greater than 10?
4. If a is a whole number, what is the next number above it?
5. Write seven numbers in order of magnitude, so that y shall be the middle number.
6. What is the sum of $y + y + y + \text{etc.}$, written b times?
7. If the product is $a b c$, the multiplier b , what is the multiplicand?
8. A man who has x dollars per day, spends a dollars per week; how many dollars has he left at the end of two weeks?
9. A regiment of men can be drawn up in a ranks of b men, and there are x men over; of how many men does the regiment consist?
10. A farmer bought b sheep and half as many horses. What will represent the whole number of both?
11. A horse cost \$12 more than four times as much as a cow, and an ox $\frac{3}{4}$ as much as the horse. What will represent the price of the ox, if the cow cost x dollars?

12. A man has four sons, each of whom is three years older than his next younger brother ; what will represent the age of the eldest, if x represents the age of the youngest ?

13. A man has a number, x , of swine, four more than three times as many cows as swine ; and his number of sheep is equal to that of his swine and cows together. What will represent the aggregate number of his whole stock ?

14. A man, having x dollars, doubled his money, then gave away \$20, and finally lost $\frac{1}{3}$ of what he then had. What will represent the number of dollars lost ?

15. A boy having a certain number, x , of cents, doubled his money, and afterward lost \$2. What will represent the cents he then had ?

16. A draper, having 50 yards of cloth worth 10 cents per yard, sold a number of yards, represented by x . What will represent the value of the yards remaining ?

17. A grocer, having 500 lbs. of coffee, sold 6 bags, each containing x lbs. What will represent the number of pounds remaining ?

18. A and B had the same number of eggs. A gave B 4 eggs, after which each broke $\frac{1}{2}$ of what he then had. What will represent the number each broke, if each had x eggs at first ?

19. If A have $3x$, and B $2y$ dollars, what will represent their money after they have lost $\frac{2}{3}$ of it ?

20. A farmer has 100 sheep in one pasture and 75 in another. What will represent the number altogether, after he has lost $\frac{1}{x}$ from each pasture ?

21. A woman at the time of her marriage was $\frac{1}{3}$ as old as her husband. The husband being x years old at the time of marriage, what will represent his wife's age 5 years before ?

22. A poulterer has x turkeys and y geese. After selling 10 turkeys for 25 geese, what will represent the number of each he then has?

23. Half of a man's life was spent in Europe, $\frac{1}{4}$ of it in Asia, and the remainder, which was 10 years, in America. Find an expression for his age.

24. A man had c horses, which he sold for c dollars apiece. With this money he bought a carriage for \$70, and also two harnesses of equal value, and had \$18 left. Express how much each harness cost.

25. If the number of hours in a day be represented by x , what will express the number of minutes in two days?

26. A farm was divided among three men. The first man received a acres, the second man b acres, the third man c times as many acres as the other two. How many acres did the farm contain?

27. A man started in business with a capital of x dollars. The first year he doubled his money; the second year he gained c times his first capital; the third year he lost \$450, and gave away $\frac{1}{2}$ of the remainder. What had he left?

28. Two men are of the same age; but if one were 18 years younger, and the other 10 years older, three times the age of the former would be the same as twice the age of the latter. What two expressions will be equal, if x represents the age of each in years?

29. A freight-car laden with c barrels of wheat, valued at n dollars apiece, met with a collision, by which $\frac{1}{x}$ of the barrels were lost; the remainder were sold for b dollars per barrel. Represent how much was lost by the accident.

NUMERICAL VALUES.

Find the numerical values of the following expressions:

When $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$, $f = 0$:

$$30. \frac{4cd}{b} + \frac{8bc}{d} - \frac{10cd}{2e}. \quad 31. c^3 + 6b^2c^2 + b^4 - 4bc^3 - 8dc.$$

$$32. \frac{8a^2 + 3b^2}{a^2b^2} + \frac{2c^2 + 3b^2}{d-b} - \frac{c^2 + d^2}{e^2}.$$

$$33. \frac{d^e}{b^e}.$$

$$35. \frac{e^c + c^a}{c^4 + 3e^2}.$$

$$34. \frac{b^c + d^c}{b^3 + d}.$$

$$36. \frac{e^c + d^c}{b^2 + e^2 + 4e}.$$

$$37. \frac{a^2f + 2ab + b^2}{a+f} - \frac{b^2 + 2bf + 4e}{f+d} + \frac{c^2 + 2d + 2ad}{b+c}.$$

$$38. \sqrt{4d^2 - 3e} + de - f. \quad 39. \frac{d^2}{b}(b - c + d).$$

When $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$:

$$40. \frac{b^2 - 2bc + c^2}{a^2 - 2ab + b^2}. \quad 41. \frac{a^4 - 4a^3c + 6a^2c^2 - 4ac^3 + c^4}{b^4 - 4b^3c + 6b^2c^2 - 4bc^3 + c^4}.$$

$$42. 7a - 2b - 3c - 4a + 5b + 4c + 2a.$$

$$43. \frac{a^2 + 2ab + b^2}{a+b} - \frac{b^2 + 2bc + c^2}{b+c} + \frac{c^2 + 2cd + d^2}{c+d}.$$

$$44. \sqrt{4c^2 + 5d^2 + e}.$$

$$46. \sqrt[3]{(2e^2 + 7b)}.$$

$$45. \sqrt{(e^2 + d^2 + c^2 - a^2)}.$$

$$47. \sqrt[4]{(2b^2 + c^2 - a)}.$$

When $x = 4$:

$$48. \sqrt{(2x+1)} - \left(x + \frac{6}{\sqrt{x}}\right) - \left(3 - \frac{x^2}{4 - \sqrt[3]{2x}}\right).$$

When $a = 0$, $b = 8$, $c = 10$, $d = 4$, $x = 6$:

$$49. b\sqrt{x^3 + cd} + \sqrt{4ax} + 4c.$$

$$50. 2c\sqrt{b^2 + x^2} - \sqrt{10c + bx - d}.$$

$$51. \frac{2c - 4d}{\sqrt{8b}} + \frac{abx}{d}. \quad 52. \frac{2x + 3b}{d - a} - \sqrt{\left(\frac{2b + \frac{1}{3}cx}{b^2}\right)}.$$

$$53. \frac{d^2x}{d} + c + 8x - \frac{b}{d}.$$

$$54. \frac{\sqrt{8b + 4d + 2c}}{\sqrt{bc + cd + dx}} - \frac{c + d + x - 2b}{dx}.$$

When $a = 6$, $b = 5$, $c = 4$, $d = 1$, $e = 0$:

$$55. 5\sqrt{ab} + b^2 - 2ab - e^2. \quad 56. 3\sqrt{c} + 2a\sqrt{2a + b - d}.$$

$$57. a\sqrt{a^2 + b^2} + 3bc\sqrt{a^2 - b^2}.$$

$$58. 3a^2b + \sqrt[3]{c^2} + \sqrt{2ac + c^2}.$$

$$59. \frac{2b + c}{3a - c} - \frac{\sqrt{5b + 3}\sqrt{c + d}}{2a + c}.$$

When $a = 8$, $b = 4$, $c = 2$, $d = \frac{1}{2}$:

$$60. \sqrt[3]{a^2} + 3ab^3 + \frac{c^6}{ad}.$$

$$61. \frac{(a^2 - b^2) \times (c^2 - d^2)}{5abcd} - \frac{c^2 + (4d)^2}{abc}.$$

$$62. (2b^3 - \sqrt[3]{a^6}) \times \frac{2\sqrt{b}}{2c} - 2d^4.$$

$$63. \frac{a}{b^2} - 3\sqrt{4cd} + 11bc^3 - \frac{a^2}{2d^3}.$$

When $a = 1$, $b = 2$, $c = 3$, $d = 4$, $m = 0$:

$$64. (2-b)(3a+4b-c) + \{ab + (3d-2c)\} - 4a(2c-3b) \\ - \{abce^2 - (3c+a)\} + \{abd - (c+d)a\}b.$$

$$65. (c^2 - a^2)(b^2 - m^2) + m\{bcd(a - b^2)d\} + 3a\{a + \\ c(d - 3a)\}.$$

$$66. \{(a-b) + (c+d)\}^2 + \{(c+m) - (b-a)\}^3 - \{(m+d) \\ + (2b-c)\}^2.$$

$$67. \sqrt{(a+c)d} + \sqrt[3]{c^2(a+b)} + \{2(d+bc)^2 + (7d - b^2c)\}^{\frac{1}{2}} \\ - (bcd + a)^{\frac{3}{2}}.$$

$$68. \frac{7(am)^{\frac{1}{2}} + 3\sqrt{d-(bd+4c)}}{\frac{1}{2}abc + (cdm)^{\frac{2}{3}}} + \frac{a^2b^2c^2 - 7d + \{d^3(a+c)\}^{\frac{1}{2}}}{\{(b-a) + a^2\}\{d - (b+m)\}} \\ - \sqrt[3]{abcd - d^2}.$$

$$69. \frac{1}{5}\{ab(a+b)\} - \frac{1}{4}\{bc(c+a)\} + \frac{1}{5}\{(ca-b)(a^2b+3)\} \\ + \frac{1}{4}\{(d+c)(1+3b-2c+d)^2\}.$$

$$70. \frac{c(a+b-c)^3 + 11\{(3a+2c)(2a-b+\frac{1}{2}d)\}}{\{(3c+b) - \sqrt{d}\}(d+c+b^2-m)} \\ + \frac{\{(a+3d)^2 - (c^3+5b) - (c+d)\}^{\frac{3}{2}}}{abm + \sqrt{dc^2-a}} \\ + \frac{(2ab+cd-bd)(d+c)}{7(d+ab^2)}.$$

SIMPLE PROBLEMS.

71. What number added to the double of itself is equal to 129?

72. A stick of timber, 35 feet long, is sawed apart, so that one part is three-fourths as long as the other. What is the length of each part?

73. A merchant has \$55 value of bank-bills, in an equal number of ones, twos, threes, and fives. How many has he of each sort?

74. A father's age is double his son's, and the sum of their ages increased by 6 is equal to 72. What is the age of each?

75. A man bought an equal number of cows, sheep, and pigs, for which he paid \$1000. He paid \$20 apiece for the cows and sheep, and \$10 apiece for the pigs. How many did he buy of each?

76. A certain number, represented by x , is divided into 2 such parts that the greater is six times the less. What are the parts?

77. Divide the number m into 4 such parts, that the second part shall be three times the first, the third part as much as the first and second, and the fourth part as much as the first, second, and third.

78. A pail which held m quarts was filled with a mixture of milk and water, and there was 4 times as much milk as water. How much was there of each?

79. At an election, there were two candidates for the office of governor. One received 3 times as many votes as the other, and his majority was 21,000. How many votes had each, and how many were polled in all?

80. James had 80 cents; after spending a part of them he found he had 3 times as many left as he had spent. How many had he spent?

81. The sum of three numbers is 156. The first, multiplied by 6, will give the second; 5 times the first subtracted from 4 times the second will give the third. What are the numbers?

82. Two men start from the same point, and travel in opposite directions. The first travels 4 times as fast as the

other. How far has each travelled when they are 90 miles apart?

83. The difference of two numbers is 15 times the less number, and 6 times the less number subtracted from the greater is 150. What are the numbers?

84. Two travellers set out at the same time from two towns 150 miles apart, and travel toward each other until they meet. How long are they on the road, if one goes 6 miles, and the other 4 miles, an hour?

85. A market-man sold 10 peaches, 18 oranges, and 16 bananas for \$3.20, selling an orange at 3 times the price of a peach, and a banana at twice the price of an orange. At what price did he sell each kind?

86. Three men, A, B, and C, built 804 rods of fence. A built 7, B 5, and C 4 rods, a day. A worked 3 times as many days as B, and B worked 5 times as many days as C. Required the number of days A worked.

87. A draper bought 16 pieces of cloth: 3 were white, 4 were black, and 9 were blue. A piece of black cost twice, and a piece of blue cost 3 times, as much as a piece of white. Required the price of a piece of black, the cost of the whole being \$190.

88. A vessel containing some water was filled up by pouring in 42 gallons, and there was then in the vessel 7 times as much as at first. How many gallons did the vessel hold?

89. Mary is 24 years older than Jane, and her age is 7 times Jane's. How old is Mary?

ADDITION.

Add together :

$$90. ab - xy + cd - m + c, 6c - 3xy + 4m - cd - 3ab, \\ 5cd - 6m + 5c + 8ab - 3xy, 5m + 6c - 3cd + 2xy - 3ab, \\ 11xy - 3m - 2c + 3ab - 7cd.$$

$$91. 5m^2x + 3xy - 7, 7xy + 3 - 8m^2x + yz, 17 - yz + 7xy \\ - 11m^2x.$$

$$92. 2ap - 3xy + 4mn, 5mn - 3xz + 7xy, 3mn - 5c^3 \\ + 2ap, -4ap - 4xy - 12mn.$$

$$93. 5x^2y - 3y^2z + 4, 7y^2z - 7m - 3, 5x^2y + 3y^2z \\ - a^2b, 6 + 7m - 7y^2z.$$

$$94. a + b + c, 3b - x + y, 5(a + b) + 3x, 7c - 3m^2n, \\ 5ab + 6b - 3y, 3(x + y) - 8c.$$

$$95. 7ax^2 - 3ab\bar{y} + 7x^2y^2 - 3\sqrt{x} + 5, 7\sqrt{x} - 3 - 7ab\bar{y} \\ - 6ax^2, 3m - 5\sqrt{a} + \bar{y} + 10ab\bar{y}, 11 - ax^2 + 5\sqrt{x} - 9x^2y^2 \\ - 7m, 2x^2y^2 + 4m - 3\sqrt{x} + 5.$$

$$96. x^3 - 3x^2y^2 - y^3 - yz + y^2, 2y^3 + 7x^2y^2 + 3y^2 - 9, \\ 4yz + 3 + 3x^3 - 5y^3 + 3x^2y^2, 2y^3 - 6x^2y^2 + 2y, -3yz \\ - x^2y^2 + 4y^2, 6 - 5y^2.$$

$$97. 8x^3 - 6ax^2 + 5a^2x - 5a^3, 3x^3 + 4ax^2 + 2a^2x \\ + 6a^3, -17x^3 + 19ax^2 - 15a^2x, 13ax^2 - 27a^2x + 18a^3, \\ 12x^3 + 3a^2x - 20a^3.$$

$$98. a + 2b + 3c, 2a - b - 2c, b - a - c, c - a - b.$$

$$99. a - 2b + 3c - 4d, 3b - 4c + 5d - 2a, 5c - 6d + 3a \\ - 4b, 7d - 4a + 5b - 4c.$$

$$100. x^3 - 4x^2 + 5x - 3, 2x^3 - 7x^2 - 14x + 5, -x^3 \\ + 9x^2 + x + 8.$$

$$101. x^4 - 2x^3 + 3x^2, x^3 + x^2 + x, 4x^4 + 5x^3, 2x^2 + 3x \\ - 4, -3x^2 - 2x - 5.$$

102. $a^3 - 3a^2b + 3ab^2 - b^3, 2a^3 + 5a^2b - 6ab^2 - 7b^3,$
 $a^3 - ab^2 + 2b^3.$

103. $2a - 4c + b, 7a - 3c + m, -9a + 6c + 3ab, +7am.$

104. $7xy - 7ez + 18ax - 14by, 3xy - 5cd + 11eg$
 $+ 14ez, 13ax + 20eg - 35cd + 18, 25xy, -15eg + 9by$
 $- 12ax.$

105. $10a^2b - 12a^3bc - 15b^2c^4 + 10, -4a^2b + 8a^3bc$
 $- 10b^2c^4 - 9 - 3a^2b - 3a^3bc + 20b^2c^4 - 3, 2a^2b + 12a^3bc$
 $+ 5b^2c^4 + 2.$

106. $13ax^2 - 10y^2 + 20, 15y^2 - 20ax + 16 + 5xy, 10$
 $+ 4ax^2 + y^2 - 10ax, -17ax^2 - 9cx^2 - 18xy + 10y^2$
 $+ 9cx^2.$

107. $3xy - 5a + 6c - 3m + 5 + xy, 12 - 2m + 7 + 5m$
 $+ 2xy - 24.$

108. $3(x + y) - 4c + 6 - 14 + 55 - 3ax, -ax + 8$
 $- 5(x + y) + 5d + 8(x + y).$

109. $5(a + x) - 3cy - 4d + 24 - 8cy, -2(a + x)$
 $+ 15cy - 3(a + x).$

110. $2x^3y^2 - 5x^2y^3 - x^2y - xy^2 + 4xy + 7x^2y^3 - 3x^2y$
 $+ 2x^3y^2 + 2xy^2 + xy + x^3y^2 - xy^2 - 2x^2y^3 + 4x^2y + xy.$

111. $2a^3x + 9a^2 - 6ay + 8 + 2a^2 - 4 + 16a^3x - 10a^2$
 $+ 7ay + 4a + 8a^2 - 20 + 6(a - x) - 8a - 20a^3x - 7(a - x)$
 $+ 8 + 2a^3x - 9a^2 + 4a + (a - x).$

112. $4\sqrt{x} + 2a^2x - 3 - \sqrt{x} - a^2x + 10y - 18 + 4\sqrt{x}$
 $- a^2x - 3\sqrt{x} - 5y + 16 - 5y + 5 - 4\sqrt{x}.$

113. $4a + 2\sqrt{x-1} + 8a^2 - 2\sqrt{x^2-1} - 12ax - 6a$
 $+ 7\sqrt{x-1} + 2a + 3\sqrt{x^2-1} - 9\sqrt{x-1} + 8ax - 8\sqrt{x^2-1}.$

114. $a + b + c - d - 2e - 2a + 3b - 4c - 5d + a - 4b$
 $+ 8d - 3e + 4c - x.$

115. $a^2x^{\frac{1}{2}} + c^{\frac{1}{2}} + 2a - 4b^2cdx - 2c^{\frac{1}{2}} - a^2x^{\frac{1}{2}} - 4a$
 $+ 8b^2cdx - 8ay - a + 4c^{\frac{1}{2}} + 6a^2x^{\frac{1}{2}}.$

In the following, use some common unit for addition :

116. $-4(a-m), 5(a-m), -14(a-m), 10(a-m).$
 117. $4(a+2b)^2, -5(a+2b)^2, +12(a+2b)^2,$
 $-20(a+2b)^2.$
 118. $ax+by+cz, bx-cy+dz, cx+dy-ez.$
 119. $4dx-5my, 3dx+7ny, 7mx+4my.$
 120. $(a+b)x, (a+c)x.$
 121. $by+dx, 4by+3dx, 4y+7x.$
 122. $2x+2xy, cx+dxy, (c+b)x+2cdxy.$
 123. $ax+6y, 6ax-3y, -2x+4y.$
 124. $(b-a)\sqrt{x}, (2a-b)\sqrt{x}.$
 125. $dx^4-bx^3+cx^2, bcx^2-acx^3+c^2x, ax^2+d-cx.$
 126. $3ax^2+2by^2-8, -ax^2+2cy^2-10, bx^2+ay^2+20.$
 127. $ax^3+bx^2+cx+d, -3ax^3+2bx^2+2bx+a,$
 $4x^3+3x^2+2x+1, 2bx^3-2ax^2-ax-b.$

SUBTRACTION

128. From $2am n + 3a^2xy + 6ay^3 + 8$ take $-2am n - 3a^2xy + 6ay^3 + 8.$

129. From $4a^2y^2z - 7xy^3 + 5az^2 - 7xy + 13m - 11$ take $3a^2y^2z + 4xy^3 - 6az^2 - 11xy - 7m - 11.$

130. From $3a - 7c + 4xy^2 - 7\sqrt{a-b^2}$ take $-11a + 7c - m^2 + 6\sqrt{a-b^2}.$

131. From $5a - 6b - 7c + 4d - 11e + 7m - 16x + y - 7z$ take $4d - 7z + 5a - 6b + m - 5c + 9x - 11y + abcd.$

132. From $2ax^2 + 3abx + 4b^2x + 12b^3$ take $ax^2 - 4abx - b^2x + bx^2 - x^3.$

133. From $6x^3 - 7x^2y + 4xy^2 - 2y^3 - 5x^2 + xy - 4y^2 - 2$ take $-8x^3 - 7x^2y + xy^2 + y^3 + 9x^2 - xy + 6y^2 - 4$.

134. From $a^6 - b^6$ take $4a^3b - 6a^2b^2 + 7ab^3$, and from the result take $2a^6 - 4a^3b + 6a^2b^2 + 6ab^3 - 2b^6$.

135. From $a^3bc - 8ab^2c - a^2c^2 + abc^2 - 6b^2c$ take $-2a^3bc - 5ab^2c + 2abc^2 - 6b^2c$.

136. The subtrahend is $8a + 4b - 5c - x$, and the minuend is $-10a - 4b - 15c + 12x$.

137. The minuend is $2ab + b^2 - 4c + 9bc$, and the subtrahend is $9ab + 8b^2 - 4c - 6bc$.

138. From $6x^2 - 4y^2 - 2z^2$ take $6x^2 + 5y^2 - 2z^2$, and from the remainder take $-3y^2 + 6z - x^2$.

139. From $4a^3 - 6a^2c + 9ac^2$ take the sum of $-2a^2c - 4a^3 + 2ac^2$ and $-2a^2c + 3a^3 - ac^2$.

140. From $2a + \frac{1}{3}b + 6$ take $\frac{1}{2}a + b + \frac{1}{6}$.

141. From $\frac{2}{3}x^2 - \frac{5}{4}xy + \frac{3}{2}y^2$ take $\frac{1}{3}x^2 - \frac{1}{4}xy - \frac{1}{2}y^2$.

142. From $\frac{2}{3}ax - \frac{1}{3}xy + \frac{2}{3}$ take $\frac{1}{3}ax + \frac{2}{3}xy - \frac{1}{3}$.

143. From $a + b - c$ take $\frac{1}{2}a - \frac{1}{2}b - \frac{1}{2}c$.

144. From $4a + 3b - 2c + 8d$ take $5d + c + a + 2b$.

145. From $12xy + 3y^2 - 17x^2 + 3\sqrt{2}$ take $-5xy + 7y^2 - 19x^2 + 2\sqrt{2}$.

In the following use some common unit of subtraction:

146. From $(2a + b + c)y$ take $(a + b)y$.

147. From $(3a + c)xy$ take $2axy + cxy$.

148. From $ay + 2by - cy$ take $ay + cy$.

149. From $(a + b)x + (b + c)y$ take $(a - b)x - (b - c)y$.

150. Subtract $3bx^2 + 5ex + 6f$ from $2ax^2 + 3bx + 5c$.

151. Subtract $2ax + by + cz + d$ from $3bx + 5ay - 2d$.

152. From $2axyz - 3by^2 - 5ac$ take $-2ac + 5axyz + 4cy^2$.

153. From $6bxy - 5cz^2 + 7$ take $-8 + 3cxy - 8dz^2$.

154. From $5ax - 4by + 3cz$ take $-2dy + 3ex + 5by$.

155. From $6a + 2b - (3a + b)$ take $2a + 4b - (4a - b)$.

USE OF PARENTHESES.

Simplify the following expressions :

156. $a - [5b - \{a - (5c - \overline{2c - b} - 4b) + 2a - (a - \overline{2b + c})\}]$.

157. $x^4 - [4x^3 - \{6x^2 - (4x - 1)\}] - (x^4 + 4x^3 + 6x^2 + 4x + 1)$.

158. $a + b + (7a - b) - (2a - 3b) - (5a + 6b)$.

159. $a - b + c - (b - a + c) + (c - a + b) - (a - c + b)$.

160. $2x - 3y - 3z - (x - y + 2z) + (x + 4y + 5z) - (z - x - y)$.

161. $2a - (2b - d) - \{a - b - (2c - 2d)\}$.

162. $(a - b) - \{-a - (b - a)\} - \{-(-\{-(-a + b) - c\} - b) - c\}$.

163. $m^2 - (c^2 - a^2) - \{-m^2 - (-2a^2)\} - \{-(-5m^2 - \{-(-a^2 - c^2 + 3m^2) - c^2\} - m^2) - 2a^2\}$.

164. $1 - (-1) - \{-(-1)\} - \{-(-\{-(-1) - 1\}) - 1\}$.

165. $a^2 + 2x - \{a^2 - (2x^2 - \{-m^2 - (a^2 + 2x - \{-m^2 - (3a^2 + 3x + 3m^2)\}) - 2m^2) - a^2\}$.

166. $(a^2bc + 3c^2) + 3a^2bc - (m + c) - \{- (4a^2bc + c) - (-3c^2 - m)\}$.

$$167. 3a - (2a + 1) + \{a - (2 - a)\} - \{-1 - (-a - \{-2 - a + (-1)\} - 2a)\}.$$

$$168. \{(am + c) - 7\} - \{-(5 - 7am + c)\} - \{-3a - (-4am - \{-c - (-9 - 5c - 4a)\} - 6) - 5am\}.$$

$$169. a - [5b - \{a - (3c - 3b) + 2c - (a - 2b - c)\}].$$

$$170. 2x - [3y - \{4x - (5y - 6x)\}].$$

$$171. a - [b + \{a - (b + a)\}].$$

$$172. 6a + [4a - \{8b - (2a + 4b) - 22b\} - 7b] - [7b + \{8a - (3b + 4a) + 8b\} + 6a].$$

$$173. 2c - (6a - b) - \{c - (5a + 2b) - (a - 3b)\}.$$

$$174. 2x - \{a - (2a - [3a - (4a - [5a - (6a - x)])])\}.$$

$$175. 25a - 19b - [3b - \{4a - (5b - 6c)\}].$$

$$176. 16 - x - [7x - \{8x - (9x - \overline{3x - 6x})\}].$$

$$177. 2a - [3b + (2b - c) - 4c + \{2a - (3b - \overline{c - 2b})\}].$$

$$178. 2a - (3b + 2c) - [5b - (6c - 6b) + 5c - \{2a - (c + 2b)\}].$$

$$179. a - [2b + \{3c - 3a - (a + b)\} + \{2a - (b + c)\}].$$

COLLECTING INTO PARENTHESES.

Express $a - b + c - d - e + m - f - r - s + v + w + x$ in brackets, preserving the order of letters.

180. Taking the terms *two* together.

181. Taking the terms *three* together.

182. Taking the terms *four* together.

183. Taking the terms *six* together.

In the following, (using the same expression as above), the asterisk is used merely to denote the position to be occupied by the given letters with reference to the brackets; the sign \pm implies here that the student is to determine which *one* of these signs is to be employed.

184. Three together, using an inner bracket after the model:
 $\{ * \pm (* \pm *) \} \pm \{ \text{etc.}$

185. Three together, using an inner bracket after the model:
 $\{ (* \pm *) \pm * \} \pm \{ \text{etc.}$

186. Four together, using an inner bracket after the model:
 $\{ * \pm (* \pm * \pm *) \} \pm \{ \text{etc.}$

187. Four together, using an inner bracket after the model:
 $\{ (* \pm * \pm *) \pm * \} \pm \{ \text{etc.}$

188. Four together, using an inner bracket after the model:
 $\{ * \pm (* \pm *) \pm * \} \pm \{ \text{etc.}$

189. Six together, using an inner bracket after the model:
 $\{ * \pm * \pm * \pm (* \pm * \pm *) \} \pm \{ \text{etc.}$

190. Six together, using an inner bracket after the model:
 $\{ (\pm * \pm * \pm * \pm *) \pm * \pm * \} \pm \{ \text{etc.}$

191. Six together, using two inner brackets after the model:
 $\{ * \pm (* \pm *) \pm * \pm (* \pm *) \} \pm \{ \text{etc.}$

Express the following in Binomials, and also in Trinomials. Before a parenthesis, use the sign which happens to be before the term which shall stand first in the parenthesis.

192. $6a + 9b - 6c + 4d - 3e + 2f.$

193. $b - x - y + 3z - 2b + c.$

194. $a^6 + 4a^5 - 2a^3 - 4a^2 - a - 1.$

195. $-3a - 2b + 6c - 4d - e - 2f.$

196. $ax - cy - dz + bz + cd + ax.$

197. $3x^4 - 2x^3y + 6x^2y^2 - 4xy^3 + y - 2y^2.$

MULTIPLICATION.

198. Multiply each of the terms of $5a^4b^2 + 7a^3b^3 - 15a^5c + 23b^2d^4 - 17bc^3d^2 - 9abodm^2$ by each of the following: $11b^3$, $-8c^3$, $5abc$, $-2bdm$.

Multiply:

199. $(8m^2 + 9mn + 10n^2)$ by mn .

200. $(x^3y^3 - x^2y^2 + xy - 7)$ by xy .

201. $(12a^3b - 6a^2b^2 + 5ab^3)$ by $12a^2b^3$.

202. $(13x^3 - 17x^2y + 5xy^2 - y^3)$ by $8xy$.

203. $(8a^3 - 6ab + 12b^2)$ by $3ab^3$.

204. $(a^{n-1}b - a^{n-2}b^2 + ab^{n-1})$ by ab .

205. $(1 + 4x - 10x^2)$ by $(1 - 6x + 3x^2)$.

206. $(x^3 - 4x^2 + 11x - 24)$ by $(x^2 + 4x + 5)$.

207. $(x^3 + 4x^2 + 5x - 24)$ by $(x^2 - 4x + 11)$.

208. $(x^3 - 7x^2 + 5x + 1)$ by $(2x^2 - 4x + 1)$.

209. $(x^3 + 6x^2 + 24x + 60)$ by $(x^3 - 6x^2 + 12x + 12)$.

210. $(x^3 - 2x^2 + 3x - 4)$ by $(4x^3 + 3x^2 + 2x + 1)$.

211. $(x^4 - 2x^3 + 3x^2 - 2x + 1)$ by $(x^4 + 2x^3 + 3x^2 + 2x + 1)$.

212. $(2x^2 + 3xy + 4y^2)$ by $(3x^2 + 4xy + y^2)$.

213. $(x + 2y - 3z)$ by $(x - 2y + 3z)$.

214. $(a^2 - ax + bx + b^2)$ by $(a + b + x)$.

215. $(a^2 + b^2 + c^2 - bc - ac - ab)$ by $(a + b + c)$.

216. $(a^2 + 4bx + 4b^2x^2)$ by $(a^2 - 4bx + 4b^2x^2)$.

217. $(a^2 - 2ab + b^2 + c^2)$ by $(a^2 + 2ab + b^2 - c^2)$.

218. $(a^2 - 2ax + 4x^2)$ by $(a^2 + 2ax + 4x^2)$.
 219. $(9a^2 + 3ax + x^2)$ by $(9a^2 - 3ax + x^2)$.
 220. $(x^4 - 2ax^2 + 4a^2)$ by $(x^4 + 2ax^2 + 4a^2)$.
 221. $(x^2 + 4xy + 5y^2)$ by $(x^3 - 3x^2y - 2xy^2 + 3y^3)$.
 222. $(a^m + b^p - 2c^n)$ by $(2a^m - 3b)$.
 223. $(2a^{3-2m}b^{n+3} + 3a^{m+1}b^{n+2} + c^p)$ by $(a^{m-1}b^{1-2m} - a^p c)$.
 224. $(x^{-3p} + 3a^m x^{-2p} - 10a^{2m} x^{-p})$ by $(a^2 x^q + 5a^{m+2} x^q + p - 2a^{2m+2} x^{q+2p})$.
 225. $(3a^{4-3m}b^m c^{m-2} + 17a^{-3}b^{m+1})$ by $(3a^{6m-2}b^{2m}c^{3-4m} - 8)$.
 226. $(a^{2n} + x^{2n})$ by $(2a^{2n} - 2x^{2n})$.
 227. $(a^{2n} + a^n x^n + x^{2n})$ by $(2a^n - 2x^n)$.
 228. $(a^{3n} - a^{2n} x^n + a^n x^{2n} - x^{3n})$ by $(3a^n + 3x^n)$.
-

THEOREMS OF DEVELOPMENT.

Expand:

229. $(\frac{1}{2}bc^2 + \frac{1}{8}b^2c^3)^2$. 231. $(\frac{3}{4}c^2 - \frac{1}{2}b^3c)^2$.
 230. $(5a^2 - 5x^3)^2$. 232. $(\frac{1}{3}a^2b - \frac{2}{3}ab)^2$.
 233. $(9a + 7bc)(9a - 7bc)$.
 234. $(4a^3 + 6b^2)(4a^3 - 6b^2)$.
 235. $(2a + \frac{1}{4})(2a - \frac{1}{4})$.
 236. $(\frac{2}{3} + 4ab)(\frac{2}{3} - 4ab)$.
 237. $(4x^2y + 3xy^2)(4x^2y - 3xy^2)$.
 238. $(\frac{2}{3}x^2 + \frac{1}{4}y^2)(\frac{2}{3}x^2 - \frac{1}{4}y^2)$.

239. $(100 + 1)^2$.
240. $(1 - 2x^3)^2$.
241. $(1 + 3a^2)(1 - 3a^2)$.
242. $(2x^2 - \frac{1}{8})^2$.
243. $(4ab^3 + 7)^2$.
244. $(x^2 + 2x - 3)^2$.
245. $(x^2 + y^2 - z^2)^2$.
246. $(x^3 - y^3 - z^3)^2$.
247. $(x^2 - 6x + 7)^2$.
248. $(x^4 - 4x^2y^2 + y^4)^2$.
249. $(x + 2y - 3z)^2$.
250. $(x^2 - 2y^2 + 5z^2)^2$.
251. $(x + 2n + 4)^2$.
252. $(c + d - 4)^2$.
253. $(a + 2b - c)^2$.
254. $(8a^2 + 6b^2 - a^2)^2$.
255. $(m + n - p - r)^2$.
256. $(2x^2 - 7x + 9)^2$.
257. $(a^3 + b^3 + c^3)^2$.
258. $(a + 3)(a + 5)$.
259. $(x + 1)(x + 2)$.
260. $(2x + 1)(2x + 3)$.
261. $(3x + y)(3x + 4y)$.
262. $(x + a)(x - b)$.
263. $(x - a)(x + b)$.
264. $(x - a)(x - b)$.
265. $(x + 3)(x - 9)$.
266. $(a + 3b)(a - 5b)$.
267. $(x - 3y)(x - 5y)$.
268. $(b + 5c)(b - 7c)$.
269. $(3 + y)(7 - y)$.
270. $(3z + x)(5z - x)$.
271. $(3a - 2xy)(2xy + 3a)$.
272. $(2a - 7)(7 + 2a)$.
273. $(x + 3)(3 - x)$.
274. $(2 + 5ay)^2$.
275. $(3a - 4x^2y^3)^2$.
276. $(x - 6)(x + 11)$.
277. $(3a - 2)(3a + 5)$.
278. $(x - 4)(x - 9)$.
279. $(x + 3)(x - 7)$.
280. $(x - 2)(x - 1)$.
281. $(2a + b)^3$.
282. $(4x - z)^3$.
283. $(3m + x)^3$.
284. $(2m^2 - 3y)^3$.

$$285. (x - a)(x + b)(x - c).$$

$$286. (1 - x)(1 + x)(1 + x^2)(1 + x^4).$$

$$287. (a - x)(a + x)(a^2 + x^2)(a^4 + x^4)(a^8 + x^8).$$

$$288. (m^8 + a^8 b^8)(m^4 + a^4 b^4)(m^2 + a^2 b^2)(m + ab)(m - ab).$$

DIVISION.

Perform divisions as follows:

$$289. (45am^4n^3 - 27am^3n^4 + 63m^3n^5y) \div 9m^3n^3.$$

$$290. (36p^2q^2rs^3 - 72p^2q^2rt^3 + 84pq^2rx^2) \div -12pq^2.$$

$$291. (42am^3r^3st^2 - 63bc^2r^3st^2 + 105r^3s^3t^2x) \div -21r^3st^2.$$

$$292. (126a^4x^2y - 30a^3b^2x^2y + 18ax^3y^2) \div 6x^2y.$$

$$293. (16a^3xy - 28a^2x^2 + 4a^2x^3) \div 4a^2x.$$

$$294. (72x^5y^6 - 36x^4y^3 - 18x^2y^2) \div 9x^2y.$$

$$295. (81m^8n^7 - 54m^5n^6 + 27m^3n^2p) \div 3m^2n^2.$$

$$296. (12x^5y^2 - 8x^4y^3 - 4x^3y^4) \div 4x^3.$$

$$297. (x^6 + 10x - 33) \div (3 + x^2 - 2x).$$

$$298. (a^8 + 2a^5m^3 - 2a^4m^4 - 2a^7m + m^8 - 2am^7 + 2a^3m^5) \div (a^3 + m^3 - a^2m - am^2).$$

$$299. (6a^4 - 10a^3m - 22a^2m^2 + 46am^3 - 20m^4) \div (4am + 3a^2 - 5m^2).$$

$$300. (4a^5 - 16a^3b^2 + 10a^2b^3 + 15ab^4 - 25b^5) \div (2a^2 - 5b^2).$$

$$301. (a^3 + b^3 + c^3 - 3abc) \div (a^2 + b^2 + c^2 - bc - ac - ab).$$

$$302. (144x^4 - 145x^2y^2 + 36y^4) \div (4x + 3y).$$

$$303. (2a^{2m} + 2a^mb^p - 4a^mc^n - 3a^mb - 3b^{p+1} + 6b^nc^n) \div (a^m + b^p - 2c^n).$$

304. $(x^3 + y^3 + z^3 - 3xyz) \div (x + y + z).$
305. $(p^2 + pq + 2pr - 2q^2 + 7qr - 3r^2) \div (p - q + 3r).$
306. $(a^8 + a^6b^2 + a^4b^4 + a^2b^6 + b^8) \div (a^4 + a^3b + a^2b^2 + ab^3 + b^4).$
307. $(x^8 + x^6y^2 + x^4y^4 + x^2y^6 + y^8) \div (x^4 - x^3y + x^2y^2 - xy^3 + y^4).$
308. $(3a^2 + 8ab + 4b^2 + 10ac + 8bc + 3c^2) \div (a + 2b + 3c).$
309. $(a^4 + 4a^2x^2 + 16x^4) \div (a^2 + 2ax + 4x^2).$
310. $(x^4 + x^2y^2 + y^4) \div (x^2 - xy + y^2).$
311. $(256x^4 + 16x^2y^2 + y^4) \div (16x^2 + 4xy + y^2).$
312. $(x^5 + x^4y - x^3y^2 + x^3 - 2xy^2 + y^3) \div (x^3 + x - y).$
313. $(ax^3 + 3a^2x^2 - 2a^3x - 2a^4) \div (x - a).$
314. $(2x^2 + xy - 3y^2 - 4yz - xz - z^2) \div (2x + 3y + z).$
315. $(2a^{3n} - 6a^{2n}b^n + 6a^nb^{2n} - 2b^{3n}) \div (a^n - b^n).$
316. $(x^{m+1} + x^my + xy^m + y^{m+1}) \div (x^m + y^m).$
317. $(x^{4n} + x^{2n}y^{2n} + y^{4n}) \div (x^{2n} + x^ny^n + y^{2n}).$
318. $(a^{m+n}b^n - 4a^{m+n-1}b^{2n} - 27a^{m+n-2}b^{3n} + 42a^{m+n-3}b^{4n}) \div (a^n b^n - 7a^{n-1}b^{2n}).$
319. $(a^{3m-2n}b^{2p}c - a^{2m+n-1}b^{1-p}c^n + a^{-n}b^{-1}c^m + a^{3m-n}b^{3p+2}c^n - a^{2m+2n-1}b^3c^{2n-1} + b^{p+1}c^{m+n-1}) \div (a^{-n}b^{-p-1} + bc^{n-1}).$

USE OF PARENTHESES INVOLVING MULTIPLICATION AND DIVISION.

Simplify :

320. $a - 2(b + 3a) - 3\{b + 2(a - b)\}.$

321. $(a + b)(b + c) - (c + d)(d + a) - (a + c)(b - d).$

$$322. 4a - [2a - \{2b(x+y) - 2b(x-y)\}].$$

$$323. (x+b)(x+c) - (a+b+c)(x+b) + a^2 + ab + b^2 + 3ax.$$

$$324. 5a - 7(b-c) - [6a - (3b+2c) + 4c - \{2a - (b+c-a)\}].$$

$$325. (x+3)^3 - 3(x+2)^3 + 3(x+1)^3 - x^3.$$

$$326. (x+y)^3 + (x+y)^2 y + (x+y)y^2 - \{3x^2 y + 5y^2 x + 2y^3\}.$$

$$327. (1+x)^3 + (1+x)^2 y + (1+x)y^2 + y^3 \\ - \{3x(x+1) + y(y+1) + 2xy + 1\}.$$

$$328. bc(b^2 - c^2) + ac(c^2 - a^2) + ab(a^2 - b^2) - (a+b+c)\{a^2(b-c) + b^2(c-a) + c^2(a-b)\}.$$

$$329. (a+b)(a-b) - \{a+b-c - (b-a-c) + (b+c-a)\}(a-b-c).$$

$$330. (x-a)(x-b)(x-c) - [bc(x-a) - \{(a+b+c)x - a(b+c)\}x].$$

$$331. (a+b)^2 - (a+b)(a-b) - \{a(2b-2) - (b^2-2a)\}.$$

Arrange, enclosed in parentheses, the coefficients of the unknown quantities:

$$332. 3(am - x + y) + 5a(x + 3y) + 2(a - y)m + 4x(a + 1).$$

$$333. (a - x + y)m + 3(m + a)x + 4(a - y) + 3(a + x)y.$$

$$334. (a + m)x - 3(am + c)xy + 2(a - cm)y^2 \text{ added to } (x + y^2)a + (c + a)xy - (b + f)y^2.$$

$$335. 3(x + y + z)am + 2c(x + z) + (y - z)ac \text{ subtracted from } 3(a - b + c)y - (2m - c)x - 3m(ax + ay - az).$$

$$336. 2a(p + xy)c - 3(m - 2xy + y^2)c - 3a(y + c) \text{ subtracted from } 11(a + b)my - 3xy(a - b + c).$$

Simplify:

$$337. 2b\{3x - 2a + 6 - [12a - (b + 9c - \overline{2x - 7a - 3})]\}.$$

$$338. a\{a(m - n) - c(p - q)\} + c\{c(-m + n) + a(-p + q)\}.$$

$$339. a(b + c)^2 + b(a + c)^2 + c(a + b)^2 + (a - b)(a + c)(b - c) \\ - (a + b)(a - c)(b - c) - (a - b)(a - c)(b + c).$$

$$340. a - (c - 2) - \left(\frac{2a + 2b}{a + b}\right).$$

$$341. 2\{b + a(2 - a)\} - \left(\frac{4a^2 - 2a + 4ab}{2a}\right).$$

$$342. 3x + \left(\frac{x + bcx + 4ax}{x}\right) - \left(\frac{4bc}{4}\right) - \left(\frac{8a^2 + 2a}{2a}\right).$$

$$343. 6a - 3\left\{\left(\frac{4ab - 2b^2}{2a - b}\right) + \left(\frac{2ab + 8}{ab + 4}\right)a\right\}.$$

$$344. a^2 + \left\{\left(\frac{8a - a^2c}{a}\right) - \left(\frac{a^3 + 4a^2c}{a}\right)\right\} + 2\left(-\frac{8ac}{-2ac}\right).$$

$$345. 2\left(\frac{x^2 + 2ax + x^2}{a + x}\right) - \left\{\frac{x^2 - 4}{-2 - x} + \right. \\ \left. \frac{4a^2 - 8ax + 4x^2}{a - x}\right\}(2 - 4a).$$

$$346. 3a\left\{\frac{(x - y)(x + y)}{x^2 - y^2} - \frac{a^2 - ab + a}{a}\right\} - \left(\frac{12a^3 - 12a^2b}{-4a}\right).$$

$$347. (x + 5a)\left(\frac{x^2 - 8ax + 15a^2}{(x - 3a)}\right) - (x - 4)\left(\frac{x^2 + 3ax + 4x + 12a}{x + 3a}\right)$$

$$- 4\{-2(a + c)(a - c) - 2c^2\}.$$

$$348. \frac{a^6 + b^6}{a^2 + b^2} - (a + b)(a - b)(a^2 + b^2) - b^2\left(\frac{a^4 - b^4}{b^2 - a^2}\right).$$

$$\begin{aligned}
 \text{349. } & 2\left(\frac{x^2 - 18x + 81}{x - 9}\right) + x\left(\frac{x^2 + x - 12}{x - 3}\right) - (x - 2)(x + 2) \\
 & + \frac{-4x(2 + x)}{x + 2}.
 \end{aligned}$$

$$\begin{aligned}
 \text{350. } & (7a - \overline{3b + 3a})\left(4a - \frac{8ab}{2b}\right) - \\
 & (a - \overline{3b - 3a})(5a + \overline{3b - a}) + \left\{\frac{(4a - x)(4a + x)}{7a - x + 3a}\right\}(4a - x).
 \end{aligned}$$

II.

FACTORING.

RESOLVE the following into prime factors.

1. $36x^6 + 24x^3y + 4y^2$.
2. $16a^2 - 9b^2$.
3. $4a^2 + 12ac + 9c^2$.
4. $x^2 - 49$.
5. $x^2 - 196$.
6. $x^2 + 12x + 36$.
7. $x^2 + 32x + 256$.
8. $x^2y^2 + 144xy + 5184$.
9. $16a^2b^2c^2 + 24ab^2c^3 + 9b^2c^4$.
10. $8c^2 - 6cd - 5d^2$.
11. $225x^2 + 420xy + 196y^2$.
12. $49x^4 - 70x^2y^2 + 25y^4$.
13. $x^4 + 4x^3 - 8x + 4$.
14. $x^4 - 10x^3 + 39x^2 - 70x + 49$.
15. $4x^4 - 12x^3 - 7x^2 + 24x + 16$.
16. $x^2 + 4y^2 + 9z^2 + 4xy + 6xz + 12yz$.
17. $x^4 + 2x^3y + x^2y^2 - y^4$.
18. $x^4 + x^2y^2 + y^4$.
19. $x^4 - x^2y^2 - 2xy^3 - y^4$.
20. $x^4 - x^2y^2 + 2xy^3 - y^4$.
21. $x^6 + 2x^4 + 5x^2 - 1$.
22. $x^4 - 18x^2 + 81$.
23. $a^4 - 4a^2b^2 - 4ab^3 - b^4$.
24. $16x^4 + 96x^3y + 144x^2y^2 - 81y^4$.
25. $a^4x^4 - b^4y^4$.
26. $a^4x^4 - 2a^2b^2x^2y^2 + b^4y^4$.
27. $(a+b)^2 - (c+d)^2$.
28. $(x+y)^2 - (x-y)^2$.
29. $x^2 - 2xy + y^2 - z^2$.
30. $(a-b)^2 - (m+n)^2$.
31. $a^2 - 2ac + c^2 - b^2 - 2bd - d^2$.

$$32. 2bc - b^2 - c^2 + a^2. \quad 33. 2xy + x^2 + y^2 - z^2.$$

$$34. 2mn - m^2 - n^2 + a^2 + b^2 - 2ab.$$

$$35. (ax + by)^2 - 1. \quad 39. x^2 - 2yz - y^2 - z^2.$$

$$36. (ax + by)^2 - (ax - by)^2. \quad 40. a^2 - 4b^2 - 9c^2 + 12bc.$$

$$37. 1 - a^2 - b^2 + 2ab. \quad 41. a^4 - 16b^2.$$

$$38. 2xy - x^2 - y^2 + 1. \quad 42. 1 - 49c^2.$$

$$43. a^2 + b^2 - c^2 - d^2 - 2ab - 2cd.$$

$$44. a^2 - b^2 + c^2 - d^2 - 2ac + 2bd.$$

$$45. 3a^3x^3 - 27ax. \quad 47. (5x - 2)^2 - (x - 4)^2.$$

$$46. a^4b^6 - c^8. \quad 48. (7x + 4y)^2 - (2x + 3y)^2.$$

$$49. 2a^2 + 7ab - 6ac + 6b^2 - 11bc + 4c^2.$$

$$50. a^3 - m^3. \quad 55. a^{11} - b^{11}.$$

$$51. a^5 + c^5. \quad 56. a^4 - m^4x^4.$$

$$52. a^4 + x^4. \quad 57. 32a^5 + x^5.$$

$$53. a^6 - b^6. \quad 58. a^6 + 2a^3b^3 + b^6.$$

$$54. a^9 - x^9. \quad 59. a^6 - 2a^3b^3 + b^6.$$

$$60. x^2 + y^2 + z^2 + 2xy + 2xz + 2yz.$$

$$61. x^2 + y^2 + z^2 - 2xy + 2xz - 2yz.$$

$$62. x^3 + 3ax^2 + 3a^2x + a^3. \quad 65. x^3 - 3x^2 + 3x - 1.$$

$$63. x^3 - 3ax^2 + 3a^2x - a^3. \quad 66. x^3 + 6x^2 + 12x + 8.$$

$$64. x^3 + 3x^2 + 3x + 1. \quad 67. a^6 - 3a^4b^2 + 3a^2b^4 + b^6.$$

$$68. a^3 + 3a^2b + 3ab^2 + b^3 + c^3 + 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2.$$

$$69. a^3 - 3a^2b + 3ab^2 - b^3 - c^3 - 3a^2c + 6abc - 3b^2c + 3ac^2 - 3bc^2.$$

70. $m^4 - 2m^2n^2 + n^4$. 95. $x^2 + mx + nx + mn$.
 71. $m^4 + 2m^3n - 2mn^3 - n^4$. 96. $y^6 - 4y^3 + 3$.
 72. $y^2 + 19ny + 48n^2$ 97. $x^2y - abx - cxy + abc$.
 73. $z^2 + 29pz + 100p^2$. 98. $x^2 + (a - b)x - ab$.
 74. $x^4 + 5x^2 + 6$. 99. $x^2 - (c - d)x - cd$.
 75. $x^6 + 4x^3 + 3$. 100. $ab^2 - bd + cd - abc$.
 76. $x^2y^2 + 18xy + 32$. 101. $4x^2 - 28xy + 48y^2$.
 77. $x^8y^4 + 7x^4y^2 + 12$. 102. $25a^3 - 30a^2b + 9ab^2$.
 78. $m^{10} + 10m^5 + 16$. 103. $9a^4 + 12a^2b + 4b^2$.
 79. $n^2 + 27nq + 140q^2$. 104. $x^2 + 2x - 63$.
 80. $n^2 - 57n + 56$. 105. $x^2 - 10x + 21$.
 81. $x^6 - 7x^3 + 12$. 106. $x^2 + 14x + 33$.
 82. $a^2b^2 - 27ab + 26$. 107. $x^2 + x - 42$.
 83. $b^4c^6 - 11b^2c^3 + 30$. 108. $x^2 - 9x + 20$.
 84. $x^2y^2z^2 - 13xyz + 22$. 109. $x^4 + 2bx^2 - a^2x^2 + b^2$.
 85. $b^2 + 25b - 150$. 110. $8a^3 - 1$.
 86. $x^8 + 3x^4 - 4$. 111. $27a^3 - 27y^3$.
 87. $x^2y^2 + 3xy - 154$. 112. $a^3 - 8b^3$.
 88. $m^{10} + 15m^5 - 100$. 113. $27y^3 + 1$.
 89. $n^2 + 17n - 390$. 114. $a^3 - 125$.
 90. $z^2 - 15z - 100$. 115. $256x^4 - 36x^2y^2 + 25y^4$.
 91. $x^{10} - 9x^5 - 10$. 116. $225x^4 - 46x^2y^2 + 49y^4$.
 92. $c^2d^2 - 24cd - 180$. 117. $289a^4 - 42a^2b^2 + 169y^4$.
 93. $m^6n^2 - m^3n - 2$. 118. $25x^4 - 36x^2y^2 + 4y^4$.
 94. $p^8q^4 - 5p^4q^2 - 84$. 119. $49a^4 - 169a^2y^2 + 144y^4$.

120. $9a^4 - 4a^2b^2 + 4b^4.$

121. $4a^2 - 11ab + 17ac - 3b^2 + bc + 4c^2.$

122. $2x^2 + ax + cx - a^2 + 10ac - 21c^2.$

123. $3x^2 + 5xy - 7cx - 2y^2 + 7cy - 6c^2.$

124. $4x^2 + 10xy + 3xz - 6y^2 - 5yz - z^2.$

125. $2a^2 - 4ab - 4ac + 2b^2 + 4bc + 2c^2.$

126. $8x^2 + 6xy + 10xz - 2y^2 + 2z^2.$

127. $2a^2 + 6ax - 18a + 4x^2 - 30x + 36.$

128. $32x^5 + 243y^5.$

134. $c^{24} + x^{24}.$

129. $a^{12} + m^{12}.$

135. $x^3 - x^2 - 4.$

130. $81 - 16c^4.$

136. $3x^3 - 14x^2 + 21x - 10.$

131. $243m^5 - 32c^5.$

137. $x^4 - 3x^2 - 4x - 3.$

132. $a^{21} + x^{21}.$

138. $5x^2 + x^4 - 2x^3 - 4x.$

133. $a^{20} + m^{20}.$

139. $a^{14} + m^{14}.$

140. $ax + ax^2 + ax^3 - 1 - x - 2x^2 - x^3 - x^4.$

141. $6a^5 + 15a^4b - 4a^3c^2 - 10a^2bc^2.$

142. $9a^3b - 27a^2bc - 6abc^2 + 18bc^3.$

GREATEST COMMON DIVISOR.

Find the G. C. D. of the following expressions:

143. $18ab^3m$ and $24a^2b^2m^3.$

144. $21a^4m^2, 18a^3m^3,$ and $15a^2m^4.$

145. $8a^2x^2y + 17amxy - 3a^2m^2x^2y$ and $5xy + 3axy - 14a^2x^2y.$

146. $x^2 + 2x - mx^2 - 2mx$ and $x^2 + 4x + 4 + ax + 2a$. $x^2 + 2$
147. $3a^2(a^2 - x^2)$ and $4a^2x^2(a - x)^2$. $a^2(a^2 - x^2)$
148. $3m^3(a^3 - m^3)(a + m)$, $4m(a^2m - m^3)^2$, and $4m^2(a^2 - m^2)(a - m)$. $m^2(a^2 - m^2)(a + m)$
149. $x^2 - 4x - 21$, $x^2 - 12x + 35$, and $x^2 + 5x - 84$. $x^2 - 7$
150. $(ax - a)^2$ and $a^2(x^2 - 3x + 2)$. $a^2(x - 1)^2$
151. $x^2 + 3x - 4$, $x^2 - 2x + 1$, and $x^2 - 1$. $x^2 - 1$
152. $x^2 - y^2$, $(x + y)^2$, and $x^2 + 3xy + 2y^2$. y
153. $x^2 - y^2$, $x^3 - y^3$, and $x^2 - 7xy + 6y^2$.
154. $x^2 - 1$, $x^3 - 1$, and $x^2 + x - 2$.
155. $1 - a^2$, $1 + a^3$, and $a^2 + 5a + 4$.
156. $x^4 - y^4$ and $x^3 - x^2y - xy^2 + y^3$.
157. $x^4 + ax^3 - a^3x - a^4$ and $x^4 + a^2x^2 + a^4$.
158. $3a^3 - 24a - 9$ and $2a^3 - 16a - 6$.
159. $x^2 + ax + a^2$ and $3(x^3 - a^3)$.
160. $21(x^4 - a^4)$, $14(x^2 + a^2)$, and $18(x^2 - a^2)$.
161. $x^2 + 9ax + 14a^2$ and $x^2 - 4a^2$.
162. $x^2 - 6ax + 9a^2$ and $x^2 + ax - 12a^2$.
163. $x^4 - 3ax^3 - 8a^2x^2 + 18a^3x - 8a^4$ and $x^3 - ax^2 - 8a^2x + 6a^3$.
164. $5a^5 + 10a^4b + 5a^3b^2$ and $a^3b + 2a^2b^2 + 2ab^3 + b^4$.
165. $6a^5 + 15a^4b - 4a^3c^2 - 10a^2bc^2$ and $9a^3b - 27a^2bc - 6abc^2 + 18b^3c^3$.
166. $8a^2b^2 - 10ab^3 + 2b^4$ and $9a^4b - 9a^3b^2 + 3a^2b^3 - 3ab^4$.

167. $x^3 + 2x^2 + 2x + 1$ and $x^3 - 2x - 1$.

168. $2x^2 - xy - 6y^2$ and $3x^2 - 8xy + 4y^2$.

169. $2x^3 - 3x^2 - 2x + 3$ and $3x^4 + 2x^3 - 2x - 3$.

170. $15x^3 + 35x^2 + 3x + 7$ and $27x^4 + 63x^3 - 12x^2 - 28x$.

171. $x^4 + 2x^2 + 9$ and $7x^3 - 11x^2 + 15x + 9$.

172. $12x^2 + 55x + 63$ and $63x^3 - 36x^2 - 343x + 196$.

173. $acx^2 + (ad + bc)x + bd$ and $a^2x^2 - b^2$.

174. $a^2x^2 - 2acxz - b^2y^2 + c^2z^2$ and $a^2x^2 + 2abxy + b^2y^2 - c^2z^2$.

175. $a^3x + (x + a)ax + x^2$ and $a^4 - x^2$.

176. $x^3 + 7x^2 - x - 7$, $x^3 + 5x^2 - x - 5$, and $x^2 - 2x + 1$.

177. $y^3 - y^2 - y + 1$, $3y^2 - 2y - 1$, and $y^3 - y^2 + y - 1$.

178. $x^3 - 4x^2 + 9x - 10$, $x^3 + 2x^2 - 3x + 20$, and $x^3 + 5x^2 - 9x + 35$.

179. $x^3 - 7x^2 + 16x - 12$, $3x^3 - 14x^2 + 16x$, and $5x^3 - 10x^2 + 7x - 14$.

180. $y^3 - 5y^2 + 11y - 15$, $y^3 - y^2 + 3y + 5$, and $2y^3 - 7y^2 + 16y - 15$.

181. $2a^4 + 3a^3x - 9a^2x^2$ and $6a^4x - 17a^3x^2 + 14a^2x^3 - 3ax^4$.

182. $6x^4 - x^3y - 3x^2y^2 + 3xy^3 - y^4$ and $9x^4 - 3x^3y - 2x^2y^2 + 3xy^3 - y^4$.

183. $6a^3b - 6a^2by - 2by^3 + 2aby^2$ and $12a^2b + 3by^2 - 15aby$.

184. $a^3 + 9a^2 + 27a - 98$ and $a^2 + 12a - 28$.

185. $8a^3b^2 - 24a^2b^3 + 24ab^4 - 8b^5$ and $12a^4 - 24a^3b + 12a^2b^2$.

186. $6a^5 + 20a^4 - 12a^3 - 48a^2 + 22a + 12$ and $a^6 + 4a^5 - 3a^4 - 16a^3 + 11a^2 + 12a - 9$.

187. $2a^3 - 2a^2b - 16ab^2 + 12b^3$ and $3a^4c - 9a^3bc - 24a^2b^2c + 54ab^3c - 24b^4c$.

188. $x^3 + x^2y + xy + y^2$ and $x^4 - y^2$.

189. $x^8 + x^6y^2 + x^2y^6 + y^8$ and $x^4 - y^4$.

190. $9x^3 + 53x^2 - 9x - 18$ and $x^2 + 11x + 30$.

191. $2x^3 + x^2 - 8x + 5$ and $7x^2 - 12x + 5$.

192. $2x^4 - 4x^3 + 8x^2 - 12x + 6$ and $3x^4 - 3x^3 - 6x^2 + 9x - 3$.

193. $20x^4 + x^2 - 1$ and $25x^4 + 5x^3 - x - 1$.

194. $ac + bd + ad + bc$ and $af + 2bx + 2ax + bf$.

195. $a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$ and $a^2 - b^2 - c^2 - 2bc$.

196. $x^3 - (2a + b)x^2 + (2ab + a^2)x - a^2b$ and $3x^2 - (4a + 2b)x + 2ab + a^2$.

197. $3a^4 - a^2b^2 - 2b^4$ and $10a^4 + 15a^3b - 10a^2b^2 - 15ab^3$.

198. $2x(x-3) + 3(x-6\frac{2}{3}) + 15$ and $2x^3 - 5x^2 - 6x + 15$.

199. $x^4 + 67x^2 + 66$ and $x^4 + 2x^3 + 2x^2 + 2x + 1$.

200. $x^4 + ax^3 - 9a^2x^2 + 11a^3x - 4a^4$ and $x^4 - ax^3 - 3a^2x^2 + 5a^3x - 2a^4$.

201. $x^4 + ax^3 + 2a^2x^2 + 3a^3x + a^4$ and $x^4 + ax^3 + 2a^2x^2 + 3a^3x + ab^2x + a^4 + a^2b^2$.

202. $15a^4 + 10a^3b + 4a^2b^2 + 6ab^3 - 3b^4$ and $6a^3 + 19a^2b + 8ab^2 - 5b^3$.

203. $15x^3 - 14x^2y + 24xy^2 - 7y^3$ and $27x^3 + 33x^2y - 20xy^2 + 2y^3$.

204. $21x^2 - 83xy - 27x + 22y^2 + 99y$ and $12x^2 - 35xy - 6x - 33y^2 + 22y$.

205. $3a^3 - 12a^2 - a^2b + 10ab - 2b^2$ and $6a^3 - 17a^2b + 8ab^2 - b^3$.

206. $x^3 + 3x^2y - 9xy^2 + 5y^3$ and $x^3 - 7x^2y + 11xy^2 - 5y^3$.

207. $6x^4 - 2x^3 - 3x^2 + 4x - 1$ and $8x^4 + 2x^3 - 4x^2 + 3x + 1$.

208. $x^4 + 2x^3 - 7x^2 + 8x - 5$ and $2x^4 + 5x^3 - 16x^2 + 14x - 15$.

209. $6x^4 - x^3 - 3x^2 - 4x - 4$ and $8x^4 - 2x^3 - 19x^2 + 3x + 10$.

210. $6x^5 - 9x^4 + 19x^3 - 12x^2 + 19x - 15$ and $4x^4 - 2x^3 + 10x^2 + x + 15$.

211. $3x^5 + 2x^4 + x^2$ and $3x^4 + 2x^3 - 3x^2 + 2x - 1$.

212. $x^5 - x^2 - x + 1$ and $x^4 - x^3 + x^2 - 2x + 1$.

213. $21x^3 + 28x^2 - 15x - 20$, $21x^3 - 28x^2 - 15x + 20$, and $14x^3 - 21x^2 - 10x + 15$.

214. $x^2 - 3xy - 10y^2$, $x^2 + 2xy - 35y^2$, and $x^2 - 8xy + 15y^2$.

LEAST COMMON MULTIPLE.

Find the L. C. M. of the following expressions:

215. $(x - x^2)^2$, $(x^2 - 1)$, $4(1 + x)x$.

216. $4(a - b)^2$, $6(a^3 - b^3)$, $6(a^3 + b^3)$, $9(a^6 - b^6)$.

217. $x^2 - 3x$, $x^2 - 10x + 21$, $x^2 - 7x$.

$$218. a^3 - x^3, a^2 + x - ax - a.$$

$$219. a^3 - 9a^2 + 26a - 24, a^3 - 8a^2 + 19a - 12.$$

$$220. 3(a^3 - b^3), 4(a - b)^3, 5(a^4 - b^4), 6(a - b)^2, (a^2 - b^2)^3.$$

$$221. x^2 + bx + b^2, 5(x - b), 6(x^3 + b^3).$$

$$222. 21(x^4 - a^4), 14(x^2 + a^2), 18(x^2 - a^2).$$

$$223. x^2 + 9ax + 14a^2, x^2 - 4a^2.$$

$$224. x^2 - 6ax + 9a^2, x^2 + ax - 12a^2.$$

$$225. x^3 - x, x^3 - 1, x^3 + 1.$$

$$226. 4(x^3 - xy^2), 20(x^3 + x^2y - xy^2 - y^3), 12(xy^2 + y^3), 12(x^2 + xy)^2, 8(x^3 - x^2y).$$

$$227. 8(a^2 - b^2), 12(a + b)^2, 20(a - b)^2.$$

$$228. 4(a + b), 6(a^2 - b^2), 8(a^3 + b^3).$$

$$229. 15(a^2b - ab^2), 21(a^3 - ab^2), 35(ab^2 + b^3).$$

$$230. x^2 - 1, x^3 + 1, x^3 - 1.$$

$$231. x^2 - 1, x^2 + 1, x^4 + 1, x^8 - 1.$$

$$232. x^2 - ax - 2a^2, x^3 + ax^2, ax^2 - a^3.$$

$$233. x^3 - x^2y - a^2x + a^2y, x^3 + ax^2 - xy^2 - ay^2.$$

$$234. x^2 - 1, x^3 + 1, x^3 - 1, x^6 + 1.$$

$$235. x^2 + 3x + 2, x^2 + 4x + 3, x^2 + 5x + 6.$$

$$236. x^2 + 2x - 3, x^3 + 3x^2 - x - 3, x^3 + 4x^2 + x - 6.$$

$$237. x^2 + 5x + 10, x^3 - 19x - 30, x^3 - 15x - 50.$$

$$238. x^2 - 3x + 2, x^2 - 4x + 3, x^2 - 5x + 4.$$

$$239. x^2 + 5x + 4, x^2 + 4x + 3, x^2 + 7x + 12.$$

$$240. x^2 - 9x + 20, x^2 - 12x + 35, x^2 - 11x + 28.$$

$$241. 6x^2 - x - 2, 21x^2 - 17x + 2, 14x^2 + 5x - 1.$$

$$242. x^2 - 1, x^2 + 2x - 3, 6x^2 - x - 2.$$

$$243. x^3 - 27, x^2 - 15x + 36, x^3 - 3x^2 - 2x + 6.$$

$$244. x^3 - 6x^2 + 11x - 6, x^3 - 9x^2 + 26x - 24, x^3 - 8x^2 + 19x - 12.$$

$$245. x^2 - 4a^2, x^3 + 2ax^2 + 4a^2x + 8a^3, x^3 - 2ax^2 + 4a^2x - 8a^3.$$

$$246. x^2 - (a+b)x + ab, x^2 - (b+c)x + bc, x^2 - (a+c)x + ac.$$

$$247. 2x^3 + (2a - 3b)x^2 - (2b^2 + 3ab)x + 3b^3, 2x^2 - (3b - 2c)x - 3bc.$$

$$248. 6(a^3 - b^3)(a - b)^3, 9(a^4 - b^4)(a - b)^2, 12(a^2 - b^2)^3.$$

$$249. x^2 + 11x + 30, x^2 + 12x + 35.$$

$$250. x^2 - 9x - 22, x^2 - 13x + 22.$$

$$251. 2x^2 + 3x + 1, x^2 - x - 2.$$

$$252. x^3 + x^2y + xy + y^2, x^4 - y^4.$$

$$253. x^2 - 8x + 15, x^2 + 2x - 15.$$

$$254. 21x^3 - 26x + 8, 7x^3 - 4x^2 - 21x + 12.$$

$$255. x^3 + x^2y + xy^2 + y^3, x^3 - x^2y + xy^2 - y^3.$$

$$256. a^3 + 2a^2b - ab^2 - 2b^3, a^3 - 2a^2b - ab^2 + 2b^3.$$

$$257. x^2 - 3x - 4, x^2 - x - 12.$$

$$258. x^3 + 5x^2 + 7x + 2, x^2 + 6x + 8.$$

$$259. 12x^2 + 5x - 3, 6x^3 + x^2 - x.$$

$$260. x^3 - 6x^2 + 11x - 6, x^3 - 9x^2 + 26x - 24.$$

$$261. x^3 - 7x - 6, x^3 + 8x^2 + 17x + 10.$$

$$262. x^4 + x^3 + 2x^2 + x + 1, x^4 - 1.$$

$$263. x^4 - 2x^3 - 3x^2 + 8x - 4, x^4 - 5x^3 + 20x - 16.$$

$$264. x^4 + a^2x^2 + a^4, x^4 - ax^3 - a^3x + a^4.$$

$$265. x^3 + 3x^2y - 9xy^2 + 5y^3, x^3 - 7x^2y + 11xy^2 - 5y^3.$$

$$266. 6x^4 - 2x^3 - 3x^2 + 4x - 1, 8x^4 + 2x^3 - 4x^2 + 3x + 1.$$

$$267. x^4 + 2x^3 - 7x^2 + 8x - 5, 2x^4 + 5x^3 - 10x^2 + 14x - 15.$$

$$268. 6x^4 - x^3 - 3x^2 - 4x - 4, 8x^4 - 2x^3 - 19x^2 + 3x + 10.$$

$$269. 6x^5 - 9x^4 + 19x^3 - 12x^2 + 19x - 15, 4x^4 - 2x^3 + 10x^2 + x + 15.$$

$$270. 3x^5 + 2x^4 + x^2, 3x^4 + 2x^3 - 3x^2 + 2x - 1.$$

$$271. x^5 - x^2 - x + 1, x^4 - x^3 + x^2 - 2x + 1.$$

$$272. 21x^3 + 28x^2 - 15x - 20, 21x^3 - 28x^2 - 15x + 20, 14x^3 - 21x^2 - 10x + 15.$$

$$273. x^2 - 3xy - 10y^2, x^2 + 2xy - 35y^2, x^2 - 8xy + 15y^2.$$

REDUCTION OF FRACTIONS.

Reduce the following fractions to lowest terms:

$$274. \frac{x^3 - b^2x}{x^2 + 2bx + b^2}.$$

$$276. \frac{6a^2 + 7ax - 3x^2}{6a^2 + 11ax + 3x^2}.$$

$$275. \frac{x^4 - a^4}{x^5 - a^2x^3}.$$

$$277. \frac{2x^3 - 16x - 6}{3x^3 - 24x - 9}.$$

$$278. \frac{9x^5 + 2x^3 + 4x^2 - x + 1}{15x^4 - 2x^3 + 10x^2 - x + 2}.$$

$$279. \frac{a^2d^2 - c^2d^2 - a^2c^2 + c^4}{4a^2d - 4acd - 2ac^2 + 2c^3}.$$

$$280. \frac{a^3 x + 2a^2 x^2 + 2ax^3 + x^4}{5a^5 + 10a^4 x + 5a^3 x^2}.$$

$$281. \frac{6ac + 10bc + 9ad + 15bd}{6c^2 + 9cd - 2c - 3d}.$$

$$282. \frac{ax^n - bx^{n+1}}{a^2 bx - b^3 x^3}.$$

$$283. \frac{2xy + 3y^2 + 2x^2 + 3xy}{8cx + 12cy - 10dx - 15dy}.$$

$$284. \frac{a^3 + b^3}{a^3 - b^3}.$$

$$290. \frac{x^3 + 2x^2 y + 3x^2 y^2}{2x^4 - 3x^3 y - 5x^2 y^2}.$$

$$285. \frac{a^6 - m^6}{(a + m)(a - m)}.$$

$$291. \frac{a^3 - 2a^2 b + 2ab^2 - b^3}{a^4 + a^2 b^2 + b^4}.$$

$$286. \frac{a^4 - m^4}{a^5 - a^3 m^2}.$$

$$292. \frac{a^4 - m^4}{a^3 - a^2 m - am^2 + m^3}.$$

$$287. \frac{7x^2 - 21x + 35}{11x^2 - 33x + 55}.$$

$$293. \frac{ac + bd + ad + bc}{am + 2bp + 2ap + bm}.$$

$$288. \frac{x^2 - 11x + 28}{x^2 - 4x - 21}.$$

$$294. \frac{x^2 + (a + b)x + ab}{x^2 + (b + c)x + bc}.$$

$$289. \frac{4x^2 + 12x + 9}{2x^2 - 5x - 12}.$$

$$295. \frac{2x^3 + x^2 - 8x + 5}{7x^2 - 12x + 5}.$$

$$296. \frac{(a + m)(a + m + x)(a + m - x)}{2a^2 m^2 + 2a^2 x^2 + 2m^2 x^2 - a^4 - m^4 - x^4}.$$

$$297. \frac{a^{12} + x^{12}}{a^{20} + x^{20}}.$$

$$300. \frac{14x^2 - 34x + 12}{9ax^2 - 39ax + 42a}.$$

$$298. \frac{a^3 + 1}{a^3 + 2a^2 + 2a + 1}.$$

$$301. \frac{10a - 24a^2 + 14a^3}{15 - 24a + 3a^2 + 6a^3}.$$

$$299. \frac{3ax^2 - 13ax + 14a}{7x^3 - 17x^2 + 6x}.$$

$$302. \frac{2ab^3 + ab^2 - 8ab + 5a}{7b^3 - 12b^2 + 5b}.$$

$$303. \frac{a^3 - 3a^2 + 3a - 2}{a^3 - 4a^2 + 6a - 4}.$$

$$309. \frac{x^3 - 2x^2 - 15x + 36}{3x^2 - 4x - 15}.$$

$$304. \frac{a^2 - a - 20}{a^2 + a - 12}.$$

$$310. \frac{3x^3 + x^2 - 5x + 21}{6x^3 + 29x^2 + 26x - 21}.$$

$$305. \frac{x^3 - 3x^2 + 4x - 2}{x^3 - x^2 - 2x + 2}.$$

$$311. \frac{x^4 - x^3 - 4x^2 - x + 1}{4x^3 - 3x^2 - 8x - 1}.$$

$$306. \frac{x^2 - 7x + 10}{2x^2 - x - 6}.$$

$$312. \frac{a^3 - 7a^2 + 16a - 12}{3a^3 - 14a^2 + 16a}.$$

$$307. \frac{x^3 + 3x^2 + 4x + 12}{x^3 + 4x^2 + 4x + 3}.$$

$$313. \frac{x^2 + 2x - 3}{x^2 + 5x + 6}.$$

$$308. \frac{x^4 - x^2 - 2x + 2}{2x^3 - x - 1}.$$

$$314. \frac{x^2 - 9x + 20}{x^2 + 6x - 55}.$$

$$315. \frac{9x^3 + 53x^2 - 9x - 18}{x^2 + 11x + 30}.$$

$$316. \frac{12ac + 20bc + 18ad + 30bd}{12c^2 + 18cd - 4c - 6d}.$$

$$317. \frac{2x^2 + 3x + 1}{x^2 - x - 2}.$$

$$318. \frac{a^2 - 3ab + ac + 2b^2 - 2bc}{a^2 - b^2 + 2bc - c^2}.$$

$$319. \frac{x^6 + x^6y^2 + x^2y + y^3}{x^4 - y^4}.$$

$$320. \frac{3x^3 - 3x^2y + xy^2 - y^3}{4x^2 - xy + 3y^2}.$$

$$321. \frac{ab + 2a^2 - 3b^2 - 4bc - ac - c^2}{9ac + 2a^2 - 5ab + 4c^2 + 8bc - 12b^2}.$$

$$322. \frac{3x^5 - 75a^4x}{2x^4 + 13a^2x^2 + 15a^4}.$$

$$323. \frac{x^4 - 1}{x^6 - 1}.$$

$$324. \frac{x^4 + x^3 + x^2 + x + 1}{x^5 - 1}.$$

$$332. \frac{20x^2 + x - 12}{12x^3 - 5x^2 + 5x - 6}.$$

$$325. \frac{x^6 + a^2 x^3 y}{x^6 - a^4 y^2}.$$

$$333. \frac{x^2 - 2ax + a^2}{x^3 - 2ax^2 + 2a^2x - a^3}.$$

$$326. \frac{x^4 + a^2 x^2 + a^4}{x^6 - a^6}.$$

$$334. \frac{2x^3 - 5x^2 - 8x - 16}{2x^3 + 11x^2 + 16x + 16}.$$

$$327. \frac{x^{m-1} y^{2n}}{x^{2m} y^{n+1}}.$$

$$335. \frac{x^3 - 3a^2 x + 2a^3}{2x^3 + ax^2 + a^2 x - 4a^3}.$$

$$328. \frac{x^2 - 10x + 21}{x^3 - 46x - 21}.$$

$$336. \frac{x^3 - 8x - 3}{x^4 - 7x^2 + 1}.$$

$$329. \frac{x^2 + 9x + 20}{x^3 + 7x^2 + 14x + 8}.$$

$$337. \frac{x^3 + a^3}{x^4 + a^2 x^2 + a^4}.$$

$$330. \frac{x^2 + x - 42}{x^3 - 10x^2 + 21x + 18}.$$

$$338. \frac{x^3 - x^2 - 7x + 3}{x^4 + 2x^3 + 2x - 1}.$$

$$331. \frac{6x^2 - 11x + 5}{3x^3 - 2x^2 - 1}.$$

$$339. \frac{3x^4 - 14x^3 - 9x + 2}{2x^4 - 9x^3 - 14x + 3}.$$

REDUCTION OF IMPROPER FRACTIONS.

Reduce to mixed or entire quantities :

$$340. \frac{9a^2 b^2 + 3ab - 4c^2 - 2c}{3ab + 2c}.$$

$$341. \frac{a^4 - x^4}{a^2 - ax + x^2}.$$

$$343. \frac{3a^2 - 12ab + y - 9a}{3a}.$$

$$342. \frac{a^5 x - 3ax^5}{a^3 - ax^2}.$$

$$344. \frac{2a^2 - 2ab + 4b^2}{a - b}.$$

$$345. \frac{m^2(m^4 - n^4) + 3m^3 - 3mn^2}{m^2(m^2 - n^2)}.$$

$$346. \frac{x^2 + 2xy + y^2 + x^3 - y^4}{x + y}.$$

$$347. \frac{5m^3 - 5p^3 + 3}{m - p}.$$

$$351. \frac{2x^2 - 6x - 1}{x - 3}.$$

$$348. \frac{1 - a - ab + a^2b}{ab - b}.$$

$$352. \frac{x^3 + ax^2 - 3a^2x - 3a^3}{x - 2a}.$$

$$349. \frac{m + ab + 5am}{m + b}.$$

$$353. \frac{x^3 - 2x^2}{x^2 - x + 1}.$$

$$350. \frac{x^2 + 3x + 2}{x + 3}.$$

$$354. \frac{x^4 + 1}{x - 1}.$$

REDUCTION OF MIXED QUANTITIES.

Reduce the following to improper fractions :

$$355. a + b + \frac{a^2 + b^2}{a - b}.$$

$$356. 2 + 3y - \frac{y - 5}{4y}.$$

$$357. y - \frac{a^2 - y^2 - c}{y}.$$

$$358. xy + mz + \frac{xyz - z^2m - 2m^2z}{z + 2m}.$$

$$359. (a + b)^2 - \frac{(a - b)^3}{a + b}.$$

$$360. a^2 + a + 1 + \frac{2}{a - 1}.$$

$$361. 3a + y - \frac{2a + xy}{x - y}.$$

$$362. 3ax - y^2 + m - \frac{3ax^2 + xy^2}{a + x}.$$

$$363. (a - 2)^3 - \frac{4a^3 - 24a^2 + 48a - 32}{a + 2}.$$

$$364. (a-1)^4 - \frac{(a-1)^4}{a}.$$

$$365. x^3 + 3xy + y^3 + \frac{9x^2y^2}{x^3 - 3xy + y^3}.$$

$$366. c + 2ab - 3ac - \frac{b^2c - 5ab^2c + a^3}{b^2 - bc}.$$

$$367. 2b + 4bc - 8ab + \frac{12ab^2 - 8ab^2c}{2b + 2ab}.$$

$$368. 4a + x - c + \frac{3ac + cx + 2xy - 2cy}{3a + x - 2y}.$$

$$369. 2a + 5x + 9y + \frac{25x^2 - 81y^2}{2a - 5x + 9y}.$$

ADDITION AND SUBTRACTION OF FRACTIONS.

Perform the operations indicated :

$$370. \frac{1-a}{1+a} + \frac{1}{1-a} + \frac{a^2}{1-a^2}.$$

$$371. \frac{1+x^2}{1-x^2} - \frac{1-x^2}{1+x^2}.$$

$$372. \frac{a}{1-a} - \frac{a^2}{(1-a)^2} + \frac{a^3}{(1-a)^3}.$$

$$373. \frac{a-b}{ab} + \frac{c-a}{ac} + \frac{b-c}{bc}.$$

$$374. \frac{1}{2(x+1)} - \frac{4}{x+2} + \frac{9}{2(x+3)}.$$

$$375. 5x - \frac{2x}{7} + \frac{5x}{9} + x^2.$$

$$376. \frac{x^3}{(x+y)^3} + \frac{y}{x+y} - \frac{xy}{(x+y)^2}.$$

$$377. \frac{m}{m+p} - \frac{p}{m-p}.$$

$$378. \frac{3}{1+2a} - \frac{4(1-5a)}{4a^2-1} - \frac{7}{2a-1}.$$

$$379. \frac{x(16-x)}{x^2-4} + \frac{2x+3}{2-x} - \frac{2-3x}{x+2}.$$

$$380. \frac{1}{a}(x+y) + \frac{1}{b}(x+y) - \left(\frac{x+y}{a} - \frac{x-y}{b} \right).$$

$$381. \frac{m+p}{(p-x)(x-m)} + \frac{p+x}{(x-m)(m-p)} + \frac{m+x}{(m-p)(p-x)}.$$

$$382. \frac{a-b}{a+b} + \frac{b-c}{b+c} - \frac{2ab-2ac}{b(a+c) + c(a+b) - b(c-b)}.$$

$$383. \frac{1}{1-x} - \frac{1}{1+x} + \frac{3}{1-2x} - \frac{3}{1+2x}.$$

$$384. \frac{m}{a(a-b)(a-c)} + \frac{m}{b(b-a)(b-c)} + \frac{m}{c(c-a)(c-b)}.$$

$$385. \frac{(x^2+y^2)^2}{xy(x-y)^2} - \frac{y}{x} - 2 - \frac{x}{y}.$$

$$386. \frac{x^2+ax+a^2}{x^3-a^3} - \frac{x^2-ax+a^2}{x^3+a^3}.$$

$$387. \frac{x^2+y^2}{xy} - \frac{x^2}{xy+y^2} - \frac{y^2}{x^2+xy}.$$

$$388. \frac{x^2-2x+3}{x^3+1} + \frac{x-2}{x^2-x+1} - \frac{1}{x+1}.$$

$$389. \frac{1}{(x-3)(x-4)} - \frac{2}{(x-2)(x-4)} + \frac{1}{(x-2)(x-3)}.$$

$$390. \frac{1}{x(x+1)} - \frac{2x-3}{x(x+1)(x+2)} + \frac{1}{x(x+2)}.$$

$$391. \frac{1-2x}{3(x^2-x+1)} + \frac{x+1}{2(x^2+1)} + \frac{1}{6(x+1)}.$$

$$392. \frac{x-y}{x^2-xy+y^2} + \frac{1}{x+y} + \frac{xy}{x^3+y^3}.$$

$$393. \frac{1}{x-y} + \frac{x-y}{x^2+xy+y^2} + \frac{xy-2x^2}{x^3-y^3}.$$

$$394. \frac{x+1}{x^2+x+1} + \frac{x-1}{x^2-x+1} + \frac{2}{x^4+x^2+1}.$$

$$395. \frac{a+b}{ax+by} + \frac{a-b}{ax-by} + \frac{2(a^2x+b^2y)}{a^2x^2+b^2y^2}.$$

$$396. \frac{a^3}{(a+b)^3} - \frac{ab}{(a+b)^2} + \frac{b}{a+b}.$$

$$397. \frac{3}{4(1-x)^2} + \frac{3}{8(1-x)} + \frac{1}{8(1+x)} - \frac{1-x}{4(1+x^2)}.$$

$$398. \frac{a^2-(b-c)^2}{(a+c)^2-b^2} + \frac{b^2-(a-c)^2}{(a+b)^2-c^2} + \frac{c^2-(a-b)^2}{(b+c)^2-a^2}.$$

$$399. \frac{1}{x^3} + \frac{1}{x^2} - \frac{1}{x} - \frac{1}{(x^2+1)^2} + \frac{x-1}{x^2+1} - \frac{3}{x^2(x^2+1)^2}.$$

$$400. \frac{1}{2(x-1)^2} - \frac{1}{4(x-1)} + \frac{1}{4(x+1)} - \frac{1}{(x-1)^2(x+1)}.$$

$$401. \frac{x^2}{4} - \frac{y^2}{6} + \frac{z^2}{8} + \frac{y^2}{4} - \frac{z^2}{6} + \frac{x^2}{8} + \frac{z^2}{4} + \frac{x^2}{6} + \frac{y^2}{8} \\ - \left(z^2 - x^2 + \frac{y^2}{2} \right).$$

$$402. \frac{3}{1-2x} - \frac{7}{1+2x} - \frac{2-10x}{4x^2-1}.$$

$$403. \frac{x}{(x-1)^2} - \frac{1}{(x-1)^2} - \frac{x^2-3}{(x^2-1)^2}.$$

$$404. \frac{1}{(x-a)(x-2a)} - \frac{1}{(x-a)(x-3a)} \\ + \frac{1}{(x-2a)(x-3a)}.$$

$$405. \frac{a^2}{(a-b)(a-c)} + \frac{b^2}{(b-c)(b-a)} - \frac{c^2}{(c-a)(c-b)}.$$

$$406. \frac{2ax-x^2}{(a-x)^2} - \frac{a^2-5ax}{(a+x)^2} - \frac{x}{(a-x)}.$$

$$407. \frac{2x}{x^2-x^2+1} - \frac{1}{x^2-x+1} - \frac{1}{x^2+x-1}.$$

$$408. \frac{1}{x^2-7x+12} + \frac{2}{x^2-4x-3} - \frac{3}{x^2-5x+4}.$$

MULTIPLICATION AND DIVISION OF FRACTIONS.

Perform the operations indicated:

$$409. \frac{a^4-b^4}{a-b} \times \frac{a^2}{ab-b^2}.$$

$$410. \frac{a^2x-x^2}{a} \times \frac{3a}{2ax-2x^2}.$$

$$411. \frac{4a^2-16b^2}{a-2b} \times \frac{5b}{8a^2-32ab+32b^2}.$$

$$412. 8 \div \left(\frac{a \cdot a-x^2}{a^3-ax^2} - 1 \right).$$

$$413. \frac{x-x^2}{3a^2} \div \frac{2ax-2ax^2}{7}.$$

$$414. \frac{2a^6x - 2x^7}{a^2x - 2ax^2 + x^3} \div \frac{a^2 + ax + x^2}{a - x}.$$

$$415. \left(x - \frac{3x}{1+x} \right) \div \frac{x(x-2)}{1-x}.$$

$$416. \frac{x^6 - y^6}{x^4 + 2x^2y^2 + y^4} \times \frac{x^2 + y^2}{x^2 - xy + y^2} \times \frac{x+y}{x^3 - y^3}.$$

$$417. \frac{x^2 + xy}{x^2 + y^2} \times \left(\frac{x}{x-y} - \frac{y}{x+y} \right).$$

$$418. 3a \times \frac{x+1}{2a} \times \frac{x-1}{a+b}.$$

$$419. \frac{a^2 - x^2}{a+b} \times \frac{a^2 - b^2}{a+x} \times \frac{a}{x(a-x)}.$$

$$420. \frac{a^2 - m^2}{my} \times \frac{a^2 + m^2}{m-a}.$$

$$421. \frac{a^2 - x^2}{3ax} \times \frac{4ax^2}{a+x}.$$

$$422. \left(\frac{x^2}{a} - 8a + \frac{12a^3}{x^2} \right) \div \left(x - \frac{2a^2}{x} \right).$$

$$423. \left(\frac{x^2}{y^3} - \frac{1}{x} \right) \div \left(\frac{x}{y^2} + \frac{1}{y} + \frac{1}{x} \right).$$

$$424. \left(\frac{x^2}{a^2} + 1 + \frac{a^2}{x^2} \right) \div \left(\frac{x}{a} - 1 + \frac{a}{x} \right).$$

$$425. 1 + \left(\frac{a-x}{a+x} \right)^2 \div 1 - \left(\frac{a-x}{a+x} \right)^2.$$

$$426. \left(\frac{x^3}{a^3} + \frac{a^3}{x^3} - 3 \left(\frac{x^2}{a^2} - \frac{a^2}{x^2} \right) + \frac{x}{a} + \frac{a}{x} \right) \div \left(\frac{x}{a} + \frac{a}{x} \right).$$

$$427. \frac{x^2 - 13x + 42}{x^2 - 5x} \times \frac{x^2 - 9x + 20}{x^2 - 6x}.$$

$$428. \frac{a^2-4}{a^2-1} \times \frac{a^2-1}{2a} \times \frac{a-2}{2+a}.$$

$$429. \frac{x^2-a^2}{x^2+bx-ax-ab} \times \frac{x^2+bx+cx+bc}{x^2+cx+dx+cd}.$$

$$430. \frac{x^2+x-12}{x^2-13x+40} \times \frac{x^2+2x-35}{x^2-7x-44}.$$

$$431. (1-a+a^2) \times \left(1 + \frac{1}{a} + \frac{1}{a^2}\right).$$

$$432. \left(\frac{a^6-x^6}{a^2-2ax+x^2} \times \frac{1}{a+x} \right) \\ + \left(\frac{a^2+ax+x^2}{a-x} \times a^2-ax+x^2 \right).$$

$$433. \frac{3a^2-3}{2(a+b)} \div \frac{x^2-1}{2a^2+2ab}.$$

$$434. \left(1 + \frac{y}{x+y} + \frac{x}{y}\right) \div \left(2 + \frac{x}{y} - \frac{x}{x+y}\right).$$

$$435. \left(\frac{a^2+b^2}{a^2-b^2} - \frac{a^2-b^2}{a^2+b^2} \right) \div \left(\frac{a+b}{a-b} - \frac{a-b}{a+b} \right).$$

$$436. \left(\frac{a}{bc} - \frac{b}{ac} - \frac{c}{ab} - \frac{2}{a} \right) \times \left(1 - \frac{2c}{a+b+c} \right).$$

$$437. \left(\frac{x^2}{a^2} + \frac{a^2}{x^2} - \frac{x}{a} - \frac{a}{x} + 1 \right) \times \left(\frac{x}{a} - \frac{a}{x} \right).$$

$$438. \left(\frac{x^2-2x+1}{x^2-5x+6} \right) \times \left(\frac{x^2-4x+4}{x^2-4x+3} \right) \times \left(\frac{x^2-6x+9}{x^2-3x+2} \right).$$

$$439. \left(\frac{1}{(a-b)^2} - \frac{1}{c^2} \right) \times \frac{a^2-b^2}{a-b-c}.$$

$$440. \frac{p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4}{p^4 - 4p^3q + 6p^2q^2 - 4pq^3 + q^4} \div \frac{p^3 + 3p^2q + 3pq^2 + q^3}{p^3 - 3p^2q + 3pq^2 - q^3}.$$

$$441. \left(a + \frac{ax}{a-x}\right) \times \left(a - \frac{ax}{a+x}\right) \div \left(\frac{a+x}{a-x} + \frac{a-x}{a+x}\right).$$

$$442. \left(a - \frac{2a}{x + \frac{1}{x}}\right) \div \left(\frac{x}{2} + \frac{1}{2x} - 1\right).$$

In the following, perform the operation as in multiplication and division of entire polynomial quantities :

$$443. \left(\frac{5x^2}{2} + 3ax - \frac{7a^2}{3}\right) \times \left(2x^2 - ax - \frac{a^2}{2}\right).$$

$$444. \left(\frac{15b^2}{a^6} - \frac{7b^4}{a^5} + \frac{6b^6}{a^4}\right) \times \left(\frac{8b^2}{a^2} - \frac{3b^4}{a}\right).$$

$$445. \left(\frac{x^2}{2} - \frac{x}{3} + \frac{1}{4}\right) \times \left(\frac{2x}{3} - \frac{1}{2}\right).$$

$$446. \left(\frac{3a^2}{2x^2} + \frac{3a}{10x} - \frac{4}{15}\right) \div \left(\frac{a}{x} - \frac{1}{3}\right).$$

$$447. \left(\frac{x^3}{3} - \frac{17x^2}{36} + \frac{x}{3} - \frac{1}{8}\right) \div \left(\frac{2x}{3} - \frac{1}{2}\right).$$

$$448. \left(5a^4 + \frac{7a^3b}{2} - \frac{107}{12}a^2b^2 + \frac{5ab^3}{6} + \frac{7b^4}{6}\right) \div \left(2a^2 - ab - \frac{b^2}{2}\right).$$

$$449. \left(x^4 - \frac{19}{6}a^2x^2 + \frac{a^3x}{3} + \frac{a^4}{6}\right) \div \left(x^2 - 2ax + \frac{a^2}{2}\right).$$

$$450. \left(\frac{x}{a} - \frac{a}{x} + \frac{y}{b} - \frac{b}{y}\right) \times \left(\frac{x}{a} - \frac{a}{x} - \frac{y}{b} + \frac{b}{y}\right).$$

$$451. \left(\frac{2x}{a} + \frac{a}{2x} \right) \times \left(\frac{2x}{a} + \frac{3a}{2x} \right).$$

$$452. \left(\frac{a^3}{b^3} + 3\frac{a}{b} + 3\frac{b}{a} + \frac{b^3}{a^3} \right) \div \left(\frac{a}{b} + \frac{b}{a} \right).$$

$$453. \left(\frac{a^2}{x^2} - \frac{ab}{2xy} + \frac{b^2}{y^2} \right) \times \left(\frac{3a^2}{x^2} - \frac{2ab}{5xy} + \frac{b^2}{y^2} \right).$$

$$454. \left(\frac{3}{4}x^5 - 4x^4 + \frac{77}{8}x^3 - \frac{43}{4}x^2 - \frac{33}{4}x + 27 \right) \\ \div \left(\frac{1}{2}x^2 - x + 3 \right).$$

$$455. \left(\frac{a^7b^2}{5} - \frac{47a^6b^3}{40} + \frac{9a^5b^4}{2} - 12a^4b^5 \right) \\ \div \left(\frac{2a^3b^2}{5} - \frac{3a^2b^3}{4} + 6ab^4 \right).$$

$$456. \left(\frac{a^2}{b^3} + \frac{2c^3d^4}{b^5} - \frac{7c^2}{2a^4b^3} \right) \times \left(\frac{a^2}{b^3} - \frac{2c^3d^4}{b^5} + \frac{7c^2}{2a^4b^3} \right).$$

$$457. \left(\frac{a^2x^3}{bd} + \frac{abx^2}{c^2d} - \frac{acx^2}{d^2} - \frac{b^2x}{cd^2} + \frac{a^2x}{bc} - \frac{a}{d} \right) \\ \div \left(\frac{ax}{c} - \frac{b}{d} \right)$$

$$458. \left(\frac{9a^2b^2}{4c^2} - \frac{25f^2m^2}{g^2} + \frac{70dfm}{g} - 49d^2 \right) \\ \div \left(\frac{3ab}{2c} + \frac{5fm}{g} - 7d \right)$$

$$459. \left(\frac{a^3c}{b^5} + \frac{a^4c}{b^4} - \frac{7a^5c}{b^3} - \frac{3a^6c}{b^2} + \frac{a^2c^3}{b^2} - \frac{2a^3c^3}{b} - a^4c^3 \right) \\ \div \left(\frac{a}{b^3} + \frac{3a^2}{b^2} + c^2 \right).$$

$$460. \left(\frac{a^2}{bc} - \frac{2a}{d} + \frac{ac}{be} + \frac{bc}{d^2} - \frac{c^2}{de} \right) + \left(\frac{a}{b} - \frac{c}{d} \right).$$

SIMPLIFICATION OF COMPLEX FRACTIONS.

Simplify the following expressions :

$$461. \frac{\frac{1}{3}(a-b)}{\frac{2}{3}a + \frac{5}{3}b}.$$

$$462. \frac{2\frac{1}{4} - \frac{3}{5}(x+2)}{1\frac{1}{3} + \frac{1}{2}(x-3)}.$$

$$463. \frac{1\frac{1}{2} - \frac{2}{5}(x-a)}{\frac{2}{3}(a+x) - \frac{2}{3}}.$$

$$464. \frac{\frac{1+2a}{1-2a} - \frac{1-2a}{1+2a}}{\frac{1-2a}{1+2a} + \frac{1+2a}{1-2a}}.$$

$$465. \frac{\frac{1}{1+a} - \frac{1}{1-a}}{\frac{1}{1-a} + \frac{1}{1+a}}.$$

$$469. \frac{\frac{(1-2m)^2 + (2m+1)^2}{(1-4m^2) - (1-2m)^2}}{\frac{(1+2m)^2 - (1-4m^2)}{(1-2m)^2 - (2m+1)^2}}.$$

$$470. \frac{x + \frac{1}{y}}{x + \frac{1}{y + \frac{1}{z}}} - \frac{1}{y(xy z + x + z)}.$$

$$471. \left(\frac{a-b}{a+b} + \frac{a+b}{a-b} \right) \div \left(\frac{a^2-b^2}{a^2+b^2} + \frac{a^2+b^2}{a^2-b^2} \right).$$

$$466. \frac{\frac{a^2+b^2}{b} - a}{\frac{1}{b} - \frac{1}{a}} \div \frac{a^3+b^3}{a^2-b^2}.$$

$$467. \frac{xy - \frac{1+x^2y^2}{xy}}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{xy}}}}.$$

$$468. \frac{\frac{a}{b + \frac{c}{d + \frac{e}{f}}}}{\frac{adf - ac}{bdf + be + cf}}.$$

$$472. \frac{\frac{3x}{2} + \frac{x-1}{3}}{\frac{13}{6}(x+1) - \frac{x}{3} - 2\frac{1}{2}}.$$

$$473. \frac{x-1 + \frac{6}{x-6}}{x-2 + \frac{3}{x-6}}.$$

$$474. \frac{3}{x+1} - \frac{2x-1}{x^2 + \frac{x}{2} - \frac{1}{2}}.$$

$$477. \frac{\frac{1}{(x+a)(x-b)} + \frac{1}{(x-a)(x+b)}}{\frac{1}{(x+a)(x+b)} + \frac{1}{(x-a)(x-b)}}.$$

$$478. \frac{\frac{a}{a-b} + \frac{b}{a+b}}{\frac{a}{a-b} - \frac{b}{a+b}}.$$

$$479. \frac{\frac{1}{2x + \frac{1}{3x + \frac{1}{4x}}}}{1}.$$

$$480. \frac{1+x+x^2}{1+\frac{1}{x}+\frac{1}{x^2}}.$$

$$484. \left(\frac{x}{1+\frac{1}{x}} + 1 - \frac{1}{x+1} \right) \div \left(\frac{x}{1-\frac{1}{x}} - x - \frac{1}{x-1} \right).$$

$$475. \frac{x-a}{x - \frac{(x-b)(x-c)}{x+a}}.$$

$$476. \frac{\frac{a}{1 + \frac{1}{a-1}}}{1 + \frac{a}{a^2 - \frac{1}{1 - \frac{a-1}{a}}}}.$$

$$481. \frac{\frac{a+b}{b} + \frac{b}{a+b}}{\frac{1}{a} + \frac{1}{b}}.$$

$$482. \frac{\frac{1}{ab} + \frac{1}{ac} + \frac{1}{bc}}{\frac{a^2 - (b+c)^2}{ab}}.$$

$$483. \frac{3}{1 + \frac{3}{1 + \frac{3}{1-x}}}.$$

$$485. \frac{1 + \frac{a-x}{a+x}}{1 - \frac{a-x}{a+x}} \div \frac{1 + \frac{a^2-x^2}{a^2+x^2}}{1 - \frac{a^2-x^2}{a^2+x^2}}.$$

$$486. \frac{1 - \frac{2xy}{(x+y)^2}}{1 + \frac{2xy}{(x-y)^2}} \div \left(\frac{1 - \frac{y}{x}}{1 + \frac{y}{x}} \right)^2.$$

$$487. \frac{\frac{m^2+n^2}{n} - m}{\frac{1}{n} - \frac{1}{m}} \times \frac{m^2 - n^2}{m^3 + n^3}.$$

$$488. \frac{x}{x-a} - \frac{x}{x+a} - \frac{\frac{x+a}{x-a} - \frac{x-a}{x+a}}{\frac{x+a}{x-a} + \frac{x-a}{x+a}}.$$

$$489. \frac{(a+b-c)^2 - d^2}{(a+b)^2 - (c+d)^2} + \frac{(b+c-a)^2 - d^2}{(b+c)^2 - (a+d)^2} \\ + \frac{(c+a-b)^2 - d^2}{(c+a)^2 - (b+d)^2}.$$

SUBSTITUTION.

Substitute in the following:

490. $(a+3)$ for x in $x^3 + 2x^2 - x - 10$.

491. $(a-b)$ for y in $y^2 + by + x$.

492. a for b in $a^4 + 2a^3b + 2a^2b^2 + 2ab^3 + b^4$.

493. $(x+1)$ for z , $(x-1)$ for a , in $z^3 + az^2 + a^2z + a^3$.

$$376. \frac{x^3}{(x+y)^3} + \frac{y}{x+y} - \frac{xy}{(x+y)^2}.$$

$$377. \frac{m}{m+p} - \frac{p}{m-p}.$$

$$378. \frac{3}{1+2a} - \frac{4(1-5a)}{4a^2-1} - \frac{7}{2a-1}.$$

$$379. \frac{x(16-x)}{x^2-4} + \frac{2x+3}{2-x} - \frac{2-3x}{x+2}.$$

$$380. \frac{1}{a}(x+y) + \frac{1}{b}(x+y) - \left(\frac{x+y}{a} - \frac{x-y}{b} \right).$$

$$381. \frac{m+p}{(p-x)(x-m)} + \frac{p+x}{(x-m)(m-p)} + \frac{m+x}{(m-p)(p-x)}.$$

$$382. \frac{a-b}{a+b} + \frac{b-c}{b+c} - \frac{2ab-2ac}{b(a+c)+c(a+b)-b(c-b)}.$$

$$383. \frac{1}{1-x} - \frac{1}{1+x} + \frac{3}{1-2x} - \frac{3}{1+2x}.$$

$$384. \frac{m}{a(a-b)(a-c)} + \frac{m}{b(b-a)(b-c)} + \frac{m}{c(c-a)(c-b)}.$$

$$385. \frac{(x^2+y^2)^2}{xy(x-y)^2} - \frac{y}{x} - 2 - \frac{x}{y}.$$

$$386. \frac{x^2+ax+a^2}{x^3-a^3} - \frac{x^2-ax+a^2}{x^3+a^3}.$$

$$387. \frac{x^2+y^2}{xy} - \frac{x^2}{xy+y^2} - \frac{y^2}{x^2+xy}.$$

$$388. \frac{x^2-2x+3}{x^3+1} + \frac{x-2}{x^2-x+1} - \frac{1}{x+1}.$$

$$389. \frac{1}{(x-3)(x-4)} - \frac{2}{(x-2)(x-4)} + \frac{1}{(x-2)(x-3)}.$$

$$390. \frac{1}{x(x+1)} - \frac{2x-3}{x(x+1)(x+2)} + \frac{1}{x(x+2)}.$$

$$391. \frac{1-2x}{3(x^2-x+1)} + \frac{x+1}{2(x^2+1)} + \frac{1}{6(x+1)}.$$

$$392. \frac{x-y}{x^2-xy+y^2} + \frac{1}{x+y} + \frac{xy}{x^3+y^3}.$$

$$393. \frac{1}{x-y} + \frac{x-y}{x^2+xy+y^2} + \frac{xy-2x^2}{x^3-y^3}.$$

$$394. \frac{x+1}{x^2+x+1} + \frac{x-1}{x^2-x+1} + \frac{2}{x^4+x^2+1}.$$

$$395. \frac{a+b}{ax+by} + \frac{a-b}{ax-by} + \frac{2(a^2x+b^2y)}{a^2x^2+b^2y^2}.$$

$$396. \frac{a^3}{(a+b)^3} - \frac{ab}{(a+b)^2} + \frac{b}{a+b}.$$

$$397. \frac{3}{4(1-x)^2} + \frac{3}{8(1-x)} + \frac{1}{8(1+x)} - \frac{1-x}{4(1+x^2)}.$$

$$398. \frac{a^2-(b-c)^2}{(a+c)^2-b^2} + \frac{b^2-(a-c)^2}{(a+b)^2-c^2} + \frac{c^2-(a-b)^2}{(b+c)^2-a^2}.$$

$$399. \frac{1}{x^3} + \frac{1}{x^2} - \frac{1}{x} - \frac{1}{(x^2+1)^2} + \frac{x-1}{x^2+1} - \frac{3}{x^2(x^2+1)^2}.$$

$$400. \frac{1}{2(x-1)^2} - \frac{1}{4(x-1)} + \frac{1}{4(x+1)} - \frac{1}{(x-1)^2(x+1)}.$$

$$401. \frac{x^2}{4} - \frac{y^2}{6} + \frac{z^2}{8} + \frac{y^2}{4} - \frac{z^2}{6} + \frac{x^2}{8} + \frac{z^2}{4} + \frac{x^2}{6} + \frac{y^2}{8} \\ - \left(z^2 - x^2 + \frac{y^2}{2} \right).$$

$$402. \frac{3}{1-2x} - \frac{7}{1+2x} - \frac{2-10x}{4x^2-1}.$$

$$403. \frac{x}{(x-1)^2} - \frac{1}{(x+1)^2} - \frac{x(x^2+3)}{(x^2-1)^2}.$$

$$404. \frac{1}{(x-a)(x-2a)} - \frac{1}{(x-a)(x-3a)} \\ + \frac{1}{(x-2a)(x-3a)}.$$

$$405. \frac{a^2}{(a-b)(a-c)} + \frac{b^2}{(b-c)(b-a)} + \frac{c^2}{(c-a)(c-b)}.$$

$$406. \frac{2ax+x^2}{(a-x)^2} - \frac{a^2+5ax}{(a+x)^2} - \frac{x}{(a-x)}.$$

$$407. \frac{2x}{x^4-x^2+1} - \frac{1}{x^2-x+1} + \frac{1}{x^2+x+1}.$$

$$408. \frac{1}{x^2-7x+12} + \frac{2}{x^2-4x+3} - \frac{3}{x^2-5x+4}.$$

MULTIPLICATION AND DIVISION OF FRACTIONS.

Perform the operations indicated:

$$409. \frac{a^4-b^4}{a+b} \times \frac{a^2}{ab-b^2}.$$

$$410. \frac{a^2x-x^3}{a} \times \frac{3a}{2ax-2x^2}.$$

$$411. \frac{4a^2-16b^2}{a-2b} \times \frac{5b}{8a^2+32ab+32b^2}.$$

$$412. 8 \div \left(\frac{a(a-x)^2}{a^3-ax^2} + 1 \right).$$

$$413. \frac{x+x^2}{3a^2} \div \frac{2ax+2ax^2}{7}.$$

$$414. \frac{2a^6x - 2x^7}{a^2x - 2ax^2 + x^3} \div \frac{a^2 + ax + x^2}{a - x}.$$

$$415. \left(x - \frac{3x}{1+x} \right) \div \frac{x(x-2)}{1-x}.$$

$$416. \frac{x^6 - y^6}{x^4 + 2x^2y^2 + y^4} \times \frac{x^2 + y^2}{x^2 - xy + y^2} \times \frac{x+y}{x^3 - y^3}.$$

$$417. \frac{x^2 + xy}{x^2 + y^2} \times \left(\frac{x}{x-y} - \frac{y}{x+y} \right).$$

$$418. 3a \times \frac{x+1}{2a} \times \frac{x-1}{a+b}.$$

$$419. \frac{a^2 - x^2}{a+b} \times \frac{a^2 - b^2}{a+x} \times \frac{a}{x(a-x)}.$$

$$420. \frac{a^2 - m^2}{my} \times \frac{a^2 + m^2}{m-a}.$$

$$421. \frac{a^2 - x^2}{3ax} \times \frac{4ax^2}{a+x}.$$

$$422. \left(\frac{x^2}{a} - 8a + \frac{12a^3}{x^2} \right) \div \left(x - \frac{2a^2}{x} \right).$$

$$423. \left(\frac{x^2}{y^3} - \frac{1}{x} \right) \div \left(\frac{x}{y^2} + \frac{1}{y} + \frac{1}{x} \right).$$

$$424. \left(\frac{x^2}{a^2} + 1 + \frac{a^2}{x^2} \right) \div \left(\frac{x}{a} - 1 + \frac{a}{x} \right).$$

$$425. 1 + \left(\frac{a-x}{a+x} \right)^2 \div 1 - \left(\frac{a-x}{a+x} \right)^2.$$

$$426. \left(\frac{x^3}{a^3} + \frac{a^3}{x^3} - 3 \left(\frac{x^2}{a^2} - \frac{a^2}{x^2} \right) + \frac{x}{a} + \frac{a}{x} \right) \div \left(\frac{x}{a} + \frac{a}{x} \right).$$

$$427. \frac{x^2 - 13x + 42}{x^2 - 5x} \times \frac{x^2 - 9x + 20}{x^2 - 6x}.$$

$$428. \frac{a^2 - 4}{a^2 - 1} \times \frac{a^2 - 1}{2a} \times \frac{a - 2}{2 + a}.$$

$$429. \frac{x^2 - a^2}{x^2 + bx - ax - ab} \times \frac{x^2 + bx + cx + bc}{x^2 + cx + dx + cd}.$$

$$430. \frac{x^2 + x - 12}{x^2 - 13x + 40} \times \frac{x^2 + 2x - 35}{x^2 - 7x - 44}.$$

$$431. (1 - a + a^2) \times \left(1 + \frac{1}{a} + \frac{1}{a^2}\right).$$

$$432. \left(\frac{a^6 - x^6}{a^2 - 2ax + x^2} \times \frac{1}{a + x} \right) \\ + \left(\frac{a^2 + ax + x^2}{a - x} \times a^2 - ax + x^2 \right).$$

$$433. \frac{3a^2 - 3}{2(a + b)} \div \frac{x^2 - 1}{2a^2 + 2ab}.$$

$$434. \left(1 + \frac{y}{x + y} + \frac{x}{y}\right) \div \left(2 + \frac{x}{y} - \frac{x}{x + y}\right).$$

$$435. \left(\frac{a^2 + b^2}{a^2 - b^2} - \frac{a^2 - b^2}{a^2 + b^2} \right) \div \left(\frac{a + b}{a - b} - \frac{a - b}{a + b} \right).$$

$$436. \left(\frac{a}{bc} - \frac{b}{ac} - \frac{c}{ab} - \frac{2}{a} \right) \times \left(1 - \frac{2c}{a + b + c} \right).$$

$$437. \left(\frac{x^2}{a^2} + \frac{a^2}{x^2} - \frac{x}{a} - \frac{a}{x} + 1 \right) \times \left(\frac{x}{a} - \frac{a}{x} \right).$$

$$438. \left(\frac{x^2 - 2x + 1}{x^2 - 5x + 6} \right) \times \left(\frac{x^2 - 4x + 4}{x^2 - 4x + 3} \right) \times \left(\frac{x^2 - 6x + 9}{x^2 - 3x + 2} \right).$$

$$439. \left(\frac{1}{(a - b)^2} - \frac{1}{c^2} \right) \times \frac{a^2 - b^2}{a - b - c}.$$

$$440. \frac{p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4}{p^4 - 4p^3q + 6p^2q^2 - 4pq^3 + q^4} \div \frac{p^3 + 3p^2q + 3pq^2 + q^3}{p^3 - 3p^2q + 3pq^2 - q^3}.$$

$$441. \left(a + \frac{ax}{a-x}\right) \times \left(a - \frac{ax}{a+x}\right) \div \left(\frac{a+x}{a-x} + \frac{a-x}{a+x}\right).$$

$$442. \left(a - \frac{2a}{x + \frac{1}{x}}\right) \div \left(\frac{x}{2} + \frac{1}{2x} - 1\right).$$

In the following, perform the operation as in multiplication and division of entire polynomial quantities:

$$443. \left(\frac{5x^2}{2} + 3ax - \frac{7a^2}{3}\right) \times \left(2x^2 - ax - \frac{a^2}{2}\right).$$

$$444. \left(\frac{15b^2}{a^6} - \frac{7b^4}{a^5} + \frac{6b^6}{a^4}\right) \times \left(\frac{8b^2}{a^2} - \frac{3b^4}{a}\right).$$

$$445. \left(\frac{x^2}{2} - \frac{x}{3} + \frac{1}{4}\right) \times \left(\frac{2x}{3} - \frac{1}{2}\right).$$

$$446. \left(\frac{3a^2}{2x^2} + \frac{3a}{10x} - \frac{4}{15}\right) \div \left(\frac{a}{x} - \frac{1}{3}\right).$$

$$447. \left(\frac{x^3}{3} - \frac{17x^2}{36} + \frac{x}{3} - \frac{1}{8}\right) \div \left(\frac{2x}{3} - \frac{1}{2}\right).$$

$$448. \left(5a^4 + \frac{7a^3b}{2} - \frac{107}{12}a^2b^2 + \frac{5ab^3}{6} + \frac{7b^4}{6}\right) \div \left(2a^2 - ab - \frac{b^2}{2}\right).$$

$$449. \left(x^4 - \frac{19}{6}a^2x^2 + \frac{a^3x}{3} + \frac{a^4}{6}\right) \div \left(x^2 - 2ax + \frac{a^2}{2}\right).$$

$$450. \left(\frac{x}{a} - \frac{a}{x} + \frac{y}{b} - \frac{b}{y}\right) \times \left(\frac{x}{a} - \frac{a}{x} - \frac{y}{b} + \frac{b}{y}\right).$$

$$451. \left(\frac{2x}{a} + \frac{a}{2x}\right) \times \left(\frac{2x}{a} + \frac{3a}{2x}\right).$$

$$452. \left(\frac{a^3}{b^3} + 3\frac{a}{b} + 3\frac{b}{a} + \frac{b^3}{a^3}\right) \div \left(\frac{a}{b} + \frac{b}{a}\right).$$

$$453. \left(\frac{a^2}{x^2} - \frac{ab}{2xy} + \frac{b^2}{y^2}\right) \times \left(\frac{3a^2}{x^2} - \frac{2ab}{5xy} + \frac{b^2}{y^2}\right).$$

$$454. \left(\frac{3}{4}x^5 - 4x^4 + \frac{77}{8}x^3 - \frac{43}{4}x^2 - \frac{33}{4}x + 27\right) \\ \div \left(\frac{1}{2}x^2 - x + 3\right).$$

$$455. \left(\frac{a^7b^2}{5} - \frac{47a^6b^3}{40} + \frac{9a^5b^4}{2} - 12a^4b^5\right) \\ \div \left(\frac{2a^3b^2}{5} - \frac{3a^2b^3}{4} + 6ab^4\right).$$

$$456. \left(\frac{a^2}{b^3} + \frac{2c^3d^4}{b^5} - \frac{7c^2}{2a^4b^3}\right) \times \left(\frac{a^2}{b^3} - \frac{2c^3d^4}{b^5} + \frac{7c^2}{2a^4b^3}\right).$$

$$457. \left(\frac{a^2x^3}{bd} + \frac{abx^2}{c^2d} - \frac{acx^2}{d^2} - \frac{b^2x}{cd^2} + \frac{a^2x}{bc} - \frac{a}{d}\right) \\ \div \left(\frac{ax}{c} - \frac{b}{d}\right)$$

$$458. \left(\frac{9a^2b^2}{4c^2} - \frac{25f^2m^2}{g^2} + \frac{70dfm}{g} - 49d^2\right) \\ \div \left(\frac{3ab}{2c} + \frac{5fm}{g} - 7d\right)$$

$$459. \left(\frac{a^3c}{b^5} + \frac{a^4c}{b^4} - \frac{7a^5c}{b^3} - \frac{3a^6c}{b^2} + \frac{a^2c^3}{b^2} - \frac{2a^3c^3}{b} - a^4c^3\right) \\ \div \left(\frac{a}{b^3} + \frac{3a^2}{b^2} + c^2\right).$$

$$460. \left(\frac{a^2}{bc} - \frac{2a}{d} + \frac{ac}{be} + \frac{bc}{d^2} - \frac{c^2}{de}\right) \div \left(\frac{a}{b} - \frac{c}{d}\right).$$

SIMPLIFICATION OF COMPLEX FRACTIONS.

Simplify the following expressions :

$$461. \frac{\frac{1}{3}(a-b)}{\frac{2}{3}a + \frac{3}{5}b}.$$

$$462. \frac{2\frac{1}{4} - \frac{3}{5}(x+2)}{1\frac{1}{3} + \frac{1}{2}(x-3)}.$$

$$463. \frac{1\frac{1}{2} - \frac{2}{5}(x-a)}{\frac{2}{3}(a+x) - \frac{2}{5}}.$$

$$464. \frac{\frac{1+2a}{1-2a} - \frac{1-2a}{1+2a}}{\frac{1-2a}{1+2a} + \frac{1+2a}{1-2a}}.$$

$$465. \frac{\frac{1}{1+a} - \frac{1}{1-a}}{\frac{1}{1-a} + \frac{1}{1+a}}.$$

$$469. \frac{\frac{(1-2m)^2 + (2m+1)^2}{(1-4m^2) - (1-2m)^2}}{\frac{(1+2m)^2 - (1-4m^2)}{(1-2m)^2 - (2m+1)^2}}.$$

$$470. \frac{x + \frac{1}{y}}{x + \frac{1}{y + \frac{1}{z}}} - \frac{1}{y(xyz + x + z)}.$$

$$471. \left(\frac{a-b}{a+b} + \frac{a+b}{a-b} \right) \div \left(\frac{a^2-b^2}{a^2+b^2} + \frac{a^2+b^2}{a^2-b^2} \right).$$

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$$466. \frac{\frac{a^2+b^2}{b} - a}{\frac{1}{b} - \frac{1}{a}} \div \frac{a^3+b^3}{a^2-b^2}.$$

$$467. \frac{xy - \frac{1+x^2y^2}{xy}}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{xy}}}}.$$

$$468. \frac{\frac{a}{b + \frac{c}{d + \frac{e}{f}}}}{\frac{adf - ac}{bdf + be + cf}}.$$

$$472. \frac{\frac{3x}{2} + \frac{x-1}{3}}{\frac{13}{6}(x+1) - \frac{x}{3} - 2\frac{1}{2}}.$$

$$473. \frac{x-1 + \frac{6}{x-6}}{x-2 + \frac{3}{x-6}}.$$

$$474. \frac{3}{x+1} - \frac{2x-1}{x^2 + \frac{x}{2} - \frac{1}{2}}.$$

$$477. \frac{\frac{1}{(x+a)(x-b)} + \frac{1}{(x-a)(x+b)}}{\frac{1}{(x+a)(x+b)} + \frac{1}{(x-a)(x-b)}}.$$

$$478. \frac{\frac{a}{a-b} + \frac{b}{a+b}}{\frac{a}{a-b} - \frac{b}{a+b}}.$$

$$479. \frac{2x + \frac{1}{3x + \frac{1}{4x}}}{1 + \frac{1}{x} + \frac{1}{x^2}}.$$

$$480. \frac{1+x+x^2}{1 + \frac{1}{x} + \frac{1}{x^2}}.$$

$$484. \left(\frac{x}{1 + \frac{1}{x}} + 1 - \frac{1}{x+1} \right) \div \left(\frac{x}{1 - \frac{1}{x}} - x - \frac{1}{x-1} \right).$$

$$475. \frac{x-a}{x - \frac{(x-b)(x-c)}{x+a}}.$$

$$476. \frac{\frac{a}{1 + \frac{1}{a-1}}}{1 + \frac{a}{a^2 - \frac{1}{1 - \frac{a-1}{a}}}}.$$

$$481. \frac{\frac{a+b}{b} + \frac{b}{a+b}}{\frac{1}{a} + \frac{1}{b}}.$$

$$482. \frac{\frac{1}{ab} + \frac{1}{ac} + \frac{1}{bc}}{\frac{a^2 - (b+c)^2}{ab}}.$$

$$483. \frac{3}{1 + \frac{3}{1 + \frac{3}{1-x}}}.$$

$$485. \frac{1 + \frac{a-x}{a+x}}{1 - \frac{a-x}{a+x}} \div \frac{1 + \frac{a^2-x^2}{a^2+x^2}}{1 - \frac{a^2-x^2}{a^2+x^2}}.$$

$$486. \frac{1 - \frac{2xy}{(x+y)^2}}{1 + \frac{2xy}{(x-y)^2}} \div \left(\frac{1 - \frac{y}{x}}{1 + \frac{y}{x}} \right)^2.$$

$$487. \frac{\frac{m^2+n^2}{n} - m}{\frac{1}{n} - \frac{1}{m}} \times \frac{m^2-n^2}{m^3+n^3}.$$

$$488. \frac{x}{x-a} - \frac{x}{x+a} - \frac{\frac{x+a}{x-a} - \frac{x-a}{x+a}}{\frac{x+a}{x-a} + \frac{x-a}{x+a}}.$$

$$489. \frac{(a+b-c)^2-d^2}{(a+b)^2-(c+d)^2} + \frac{(b+c-a)^2-d^2}{(b+c)^2-(a+d)^2} \\ + \frac{(c+a-b)^2-d^2}{(c+a)^2-(b+d)^2}.$$

SUBSTITUTION.

Substitute in the following:

490. $(a+3)$ for x in $x^3 + 2x^2 - x - 10$.

491. $(a-b)$ for y in $y^2 + by + x$.

492. a for b in $a^4 + 2a^3b + 2a^2b^2 + 2ab^3 + b^4$.

493. $(x+1)$ for z , $(x-1)$ for a , in $z^3 + az^2 + a^2z + a^3$.

494. $(y + 1)$ for a in $a^4 - 16ay + 2ab$.

495. 2 for $(a + b)$ in $(c + a + b)^5 + (c - a - b)^5$.

496. $(y + 1)^2$ for x , $(y - 1)$ for a , in $2x^2 - 4a^3x + 16a$.

497. $(x + 2)$ for x in $x^3 - 3x^2 + 3x - 1$.

498. $(x + 1)$ for x , $(y + 1)$ for y , in $x^2 + 2xy + y^2$.

499. $(a + 1)$ for a , $(a + 2)$ for x , $(a + 3)$ for b , in $a^2x + bx^2 + a^2b^3$.

500. $(a + x)$ for $(a + b)$ in $ax + bx + a^2 - b^2$.

501. $(2a + x)$ for x in $ax^3 - 4x^2 + 2ax$.

502. $(m - 1)$ for a , $(m + 1)$ for m , in $2am + 3a^2m^2 - 4am^2$.

503. $(1 + a)$ for x , $(x + a)$ for a , in $2x^4 - 7x^3 + ax^2$.

504. $(2 + x)$ for a , $(2 - y)$ for x , $(2 + a)$ for y , in $4a^2 - 4axy + x^2y^2$.

Find the values of the following expressions:

505. $\frac{a - x}{b - x}$ when $x = \frac{ab}{a + b}$.

506. $\frac{x - a}{b} - \frac{x - b}{a}$ when $x = \frac{a^2}{a - b}$.

507. $\frac{x}{a} + \frac{x}{b - a} - \frac{a}{a + b}$ when $x = \frac{a^2(b - a)}{b(b + a)}$.

508. $\frac{a^2x + b^2y}{x + y}$ when $a = \frac{2}{3}$ and $b = \frac{2}{3}$.

509. $\frac{x}{x + y} + \frac{y}{x - y} - \frac{y^2}{x^2 - y^2}$ when $y = \frac{3x}{4}$.

510. $\frac{x + 2a}{2b - x} + \frac{x - 2a}{2b + x} - \frac{4ab}{4b^2 - x^2}$ when $x = \frac{ab}{a + b}$.

$$511. \left(\frac{x-a}{x-b} \right)^3 - \frac{x-2a+b}{x+a-2b} \text{ when } x = \frac{a+b}{2}.$$

$$512. \frac{x+y-1}{x-y+1} \text{ when } x = \frac{a+1}{ab+1} \text{ and } y = \frac{ab+a}{ab+1}.$$

$$513. (a+b)(b+c) - (c+d)(d+a) - (a+c)(b-d) \\ \text{when } b=d.$$

$$514. b^2 + c^2 + 2bc - a^2 \text{ when } a = y + z - 2x, \ b = z + x - 2y, \text{ and } c = x + y - 2z.$$

III.

EQUATIONS OF FIRST DEGREE: ONE UNKNOWN QUANTITY.

Find the value of x in the following equations:

1. $25x + 20 - 7x - 5 = 56 - 5x + 5.$

2. $12x - 8 - 8x + 6 - 12 + 3x = 0.$

3. $7x + 20 - 3x = -50 + 4x + 60 + 8x.$

4. $2x^2 + 13x = 13x^2 - 9x.$

5. $12x + 81 - 27x - 10x - 4 = 61 - 8x - 20 - 29 - 4x.$

6. $20x - 12 - 192 + 64x - 36x + 12 = 96.$

7. $18x + 6 + 20x + 4 - 27x - 9 = 24.$

8. $18x + 42 - 8x + 28 + 231 = 21x - 84.$

9. $bx + ax = abc.$

10. $5(x + 1) + 6(x + 2) = 9(x + 3).$

11. $4(3x - 2) - 2(4x - 3) - 3(4 - x) = 0.$

12. $7(5x + 3) - 3(16 - 5x) = 21(37 - 4x).$

13. $5x - [8x - 3\{16 - 6x - (4 - 5x)\}] = 6.$

14. $(x - 5)(x - 6) = (x - 2)(x - 3).$

15. $(x + 9)(x - 5) = (x - 7)(x + 3).$

16. $(x - a)(2x - b)^2 = (x - b)(2x - a)^2.$

17. $(3x + 1)(4x + 7) = 12(x + 1)^2.$

$$18. am - 5x = bc - ax.$$

$$19. abc - a^2x = ax - a^2b.$$

$$20. 3acx - 6bcd = 12cdx + abc.$$

$$21. \frac{x+1}{3} - \frac{x-1}{4} + 4x = 12 + \frac{2x-1}{6}.$$

$$22. \frac{4x-7}{8} + 2\frac{3}{4} + \frac{7-4x}{4} = x + \frac{13}{24}.$$

$$23. \frac{x+3}{2} - \frac{x-2}{3} = \frac{3x-5}{12} + \frac{1}{4}.$$

$$24. \frac{1}{6}(8-x) + x - 1\frac{2}{3} = \frac{1}{2}(x+6) - \frac{x}{3}.$$

$$25. \frac{3x-1}{5} - \frac{13-x}{2} = \frac{7x}{3} - \frac{11}{6}(x+3).$$

$$26. \frac{2-x}{3} + \frac{3-x}{4} + \frac{4-x}{5} + \frac{5-x}{6} + \frac{3}{4} = 0.$$

$$27. \frac{5x-3}{7} - \frac{9-x}{3} = \frac{5x}{2} + \frac{19}{6}(x-4).$$

$$28. \frac{bx+4a}{4} - \frac{a^2-3bx}{a} - bx = ab^2 - \frac{5a^2-6bx}{2a} + ax.$$

$$29. ax - bc = \frac{b^2x}{b-a}.$$

$$30. \frac{11a-3x}{a+b} - \frac{6a-5x}{a-b} = \frac{a+b}{a-b} + \frac{2x}{a^2-b^2}.$$

$$31. \frac{(a+x)^2}{4} - abx = \frac{1}{4}x^2.$$

$$32. \frac{abc}{\frac{1}{3}(a+b)} - \frac{bx}{a} + \frac{a^2b^2}{(a+b)^3} = 3cx - \frac{b^2x}{a} \cdot \frac{2a+b}{(a+b)^2}.$$

$$\begin{aligned} 33. \quad & \frac{2}{5}\left(x - \frac{1}{3}\right) + \frac{1}{3}\left\{1 - \left(x + \frac{2}{5}\right)\right\} - \frac{2}{7}\left\{x - \left(1 + \frac{1}{3}x\right)\right\} \\ & = x + \frac{2}{7}x. \end{aligned}$$

$$34. \quad \frac{8ax - b}{5} - \frac{5b}{3} = 4 - b - \frac{7c}{9}.$$

$$35. \quad (a^2 - x)(b^2 + x) - 3ab(1 - x) = (x - a)(c - x).$$

$$36. \quad \frac{5x + 3}{x - 1} + \frac{2x - 3}{2x - 2} = 9.$$

$$37. \quad \frac{3x + 1}{5} - \frac{7x + 2}{10} + \frac{3x}{4} - \frac{7x}{8} = -12.$$

$$38. \quad x - 3 - (3 - x)(x + 1) = x(x - 3) + 8.$$

$$39. \quad 3 - x - 2(x - 1)(x + 2) = (x - 3)(5 - 2x).$$

$$40. \quad (x - a)(x - b) = (x - a - b)^2.$$

$$41. \quad (a - b)(x - c) - (b - c)(x - a) - (c - a)(x - b) = 0.$$

$$42. \quad (x + 1)(x + 2)(x + 3) = (x - 1)(x - 2)(x - 3) + 3(4x - 2)(x + 1).$$

$$43. \quad (x - 9)(x - 7)(x - 5)(x - 1) = (x - 2)(x - 4)(x - 6)(x - 10).$$

$$44. \quad (8x - 3)^2(x - 1) = (4x - 1)^2(4x - 5).$$

$$45. \quad 14 - x - 5(x - 3)(x + 2) + (5 - x)(4 - 5x) = 45x - 76.$$

$$46. \quad 5(x - 2)^2 + 7(x - 3)^2 = (3x - 7)(4x - 19) + 42.$$

$$47. \quad (3x - 17)^2 + (4x - 25)^2 - (5x - 29)^2 = 1.$$

$$48. \quad am - b - \frac{ax}{b} + \frac{x}{m} = 0.$$

$$49. \quad \frac{ax^2}{b - cx} + a + \frac{ax}{c} = 0.$$

$$50. \frac{a(d^2 + x^2)}{dx} = ac + \frac{ax}{d}.$$

$$51. \frac{ab}{x} = bc + d + \frac{1}{x}.$$

$$52. c = a + \frac{m(a-x)}{3a+x}.$$

$$53. (a+x)(b+x) - a(b+c) = \frac{a^2c}{b} + x^2.$$

$$54. \frac{4}{x-8} + \frac{3}{2x-16} - 1\frac{5}{24} = \frac{2}{3x-24}.$$

$$55. x^2 - \frac{4x^2 - 20x + 24}{x^2 - 2x + 4} = x^2 + 2x - 4.$$

$$56. \frac{2x^4 + 2x^3 - 9x^2 + 12}{x^2 + 3x - 4} = 2x^2 - 4x - 3.$$

$$57. 5 - x\left(3\frac{1}{2} - \frac{2}{x}\right) = \frac{1}{2}x - \frac{3x - (4 - 5x)}{4}.$$

$$58. \frac{8x+5}{14} + \frac{7x-3\frac{1}{2}}{6x+2} = \frac{16x+15}{28} + \frac{2\frac{1}{4}}{7}.$$

$$59. \frac{ax+b}{c} + \frac{ax+b}{cx+b} = \frac{2ax+d}{2c} + \frac{b}{c}.$$

$$60. \frac{7x+5}{3} - \left(2x - \frac{3x-7}{14}\right) = 5.$$

$$61. 4x - \frac{x-2}{2} - \left\{ 2x - \left(\frac{\frac{16 - \frac{x+4}{2}}{3}}{6} \right) \right\} = \frac{3x+6}{2}.$$

$$62. \frac{7x + \frac{13}{2}}{10} + \frac{11x - \frac{x-\frac{3}{2}}{2}}{12} = \frac{3x+1}{5} + \frac{43x - \frac{3-8x}{2}}{22}.$$

$$63. \frac{2(3-4x)}{3-x} + \frac{3}{1-x} = 8.$$

$$64. \frac{15 \div 3x}{x+1} + \frac{30 \div 4x}{x+3} = 7 + \frac{24}{x+1}.$$

PROBLEMS LEADING TO EQUATIONS CONTAINING ONE UNKNOWN QUANTITY OF THE FIRST DEGREE.

65. A person buys some tea at 36 cts. a pound, and some at 60 cts. a pound; he wishes to mix them so that by selling the mixture at 44 cts. a pound he may gain 10 per ct. on each pound. Find how many pounds of the inferior tea he must mix with each pound of the superior.

66. Divide the number a into three parts, such that the second shall be m times the n th part of the first, and that the third shall be the q th part of p times the first.

67. Gold weighs $19\frac{1}{4}$ times as much as water; silver weighs $10\frac{1}{2}$ times as much as water; one cubic foot of water weighs 62 $\frac{1}{2}$ lbs. A goldsmith offers a mass of $\frac{1}{4}$ of a cubic foot, which he asserts to be gold, weighing $257\frac{1}{8}$ lbs. If alloyed, what is the proportion of silver to gold?

68. A piece of work can be finished by 4 men in 9 days, or by 10 women in 7 days, or by 15 children in 8 days. In what time can 1 man, 3 women, and 4 children finish the work?

69. A nurseryman has an orchard to plant with a given number of trees, and he finds that when he has as many rows as trees in a row there are 75 trees remaining; but if he puts 5 trees less in a row, and increases the number of rows by 7, he then has only 10 trees remaining. What is the number of trees?

70. A and B set out together from the same place and travelled in the same direction. A travelled uniformly 18 miles a day, but after 9 days turned, and went back as far as B had travelled during those 9 days; he then turned again, pursuing his journey, and overtook B in $22\frac{1}{2}$ days from the time they first set out. At what rate did B travel?

71. What must be the value of n in order that $\frac{2a + n}{3n + 69a}$ may be equal to $\frac{1}{33}$ when a is $\frac{1}{3}$?

72. Two persons, A and B, have the same annual income. A saves $\frac{1}{3}$ of his; but B, by spending \$80 per annum more than A, at the end of 4 years finds himself \$220 in debt. What is their annual income?

73. A steamer makes a down-trip from the head of Lake Ontario to Montreal in 28 hours, the current being in its favor. When returning, it is found that in ascending the St. Lawrence (three-sevenths of the entire trip) the rate of sailing is 5 miles per hour less than the average rate in its downward journey; but upon entering the lake it is enabled to increase its speed two miles per hour, and again reaches Hamilton, at the head of the lake, in $\frac{1}{2}\frac{2}{3}$ of the time it would have required from Montreal had the rate uniformly been the same as when ascending the river. Required the distance between Montreal and Hamilton, and the rates of sailing.

74. A general, ranging his army in the form of a solid square, finds he has 284 men to spare, but increasing the side by one man, he wants 25 to fill up the new square. How many soldiers has he?

75. A person observes the discharge of a gun at a distance, and hears the report exactly $10\frac{1}{2}$ seconds afterward. Assuming that light travels at the rate of 192,000 miles and sound 1090 feet per second, what is the distance between him and the gun?

76. From the 8th to the 19th of June last year the thermometer was observed to ascend $1\frac{1}{2}$ degrees each day, and the mean of all observations was 74 degrees. What was the height of the thermometer on the 8th of June?

77. In a certain grist-mill there are three runs of stones, the first of which can empty the granary in 72 hours, the second in 84 hours, and the third in 90 hours. Two teams are engaged drawing wheat and storing it in the granary; and of these, the first can fill it in 60 hours, the second in 78 hours. Now, if the granary be full, and both teams and all three runs of stones be set in operation, in what time will it be emptied?

78. The fore-wheel of a wagon is a feet, and the hind-wheel b feet in circumference. Through what distance must the wagon pass, in order that the fore-wheel shall have made n revolutions more than the hind-wheel?

79. A merchant drew every year, upon the stock he had in trade, the sum of a dollars for family expenses. His profits each year were the n th part of what remained after this deduction, but at the end of three years he found his whole stock exhausted. How much had he at the beginning?

80. From the first of two mortars in a battery 36 shells are thrown before the second is ready for firing. Shells are then thrown from both, in the proportion of 8 from the first to 7 from the second, the second mortar requiring as much powder for 3 charges as the first does for 4. How many balls must the second mortar throw in order that both shall have consumed the same quantity of powder?

81. Divide the number a into two such parts that the one shall be $\frac{n}{m}$ ths of the other.

82. A merchant bought two kinds of wine in equal quantities, giving for one m shillings a gallon, and for the other n shillings a gallon. By mixing them, and selling the mixture

at r shillings a gallon, he gained a shillings. How many gallons of each did he buy?

83. A person being asked the hour, answered that it was between 5 and 6, and the hour and minute-hand were together. What was the time?

84. A company of foot are 1165 of their own paces ahead of a troop of horse. The foot take 5 paces to every 4 of the horse, but 3 paces of the horse are equal in extent to 4 paces of the foot. How many paces will the horse march before they overtake the foot?

85. The distance from M to L is $31\frac{1}{2}$ miles. The express down train leaves M at 11.30 A. M., and arrives at L at 12.30. The up train leaves L at 11.45 A. M., and arrives at M at 12.35. Supposing the speed of each to be uniform, find where they will meet.

86. A person has a hours at his disposal. How far may he ride in a coach which goes b miles an hour, so as to return home in time, walking back at the rate of c miles an hour?

87. A farmer had sufficient food to support p cows through the season. After feeding them 9 days the cattle plague broke out, and carried off r of them daily for 10 days. In consequence also of a long-continued frost, he was obliged to feed the remaining stock three days longer than he had expected; then the food was exhausted. How many days would the food have lasted had no deaths occurred?

88. Potatoes are sold so as to gain 25 per ct. at six pounds for 10 cts. Find the gain per ct. when they are sold five pounds for 12 cts.

89. A man and a boy engaged to pull a field of turnips for \$2.52. But when $\frac{2}{3}$ of the work was done the boy ran away, and the man finished it alone. In consequence, the work occupied $1\frac{1}{4}$ days more than was expected. The boy could do

only half a man's work, and was paid in proportion. What did each earn per day?

90. An officer can form his men into a hollow square 4 deep, and also into a hollow square 8 deep. The front in the latter formation contains 16 men fewer than in the former formation. Find the number of men he has.

91. Suppose two steamboats to start at the same time from places 300 miles apart on the same river: the one proceeding up stream is retarded by the current two miles per hour, the other moving down stream is accelerated likewise. If each is propelled by a steam-engine that would move it 8 miles per hour in still water, how far from each starting-place will the boats meet?

92. A person walked out from Cambridge to a village at the rate of 4 miles an hour; and on reaching the railway-station to return, had to wait ten minutes for a train for Cambridge, which was then $4\frac{1}{2}$ miles off. Arriving at his rooms, which were a mile beyond Cambridge station, he found he had been out $3\frac{1}{4}$ hours. Find the distance of the village from his room.

93. A train 66 yds. long, which had come 50 miles from the terminus in one hour, met another train 110 yds. long, which it passed in 5 seconds. At what rate was the latter moving, and where did it meet a freight-train moving at the rate of 20 miles an hour, which left the terminus half an hour after the first?

SIMULTANEOUS EQUATIONS OF THE FIRST DEGREE: TWO UNKNOWN QUANTITIES.

Find the values of the unknown quantities:

$$94. \begin{cases} \frac{x}{6} + \frac{y}{5} = \frac{x}{2} + 2, \\ \frac{x}{4} + \frac{y}{3} = \frac{3y}{10} + 4. \end{cases} \quad 95. \begin{cases} \frac{4}{5+y} = \frac{5}{12+x}, \\ 2x+5y=35. \end{cases}$$

$$96. \begin{cases} 2x+6:3y+2::8:7, \\ 8x-4=9y. \end{cases}$$

$$97. \begin{cases} x+y-8=0, \\ \frac{x-y}{2} + \frac{2x-3}{3} + \frac{4}{3} = 0. \end{cases}$$

$$98. \begin{cases} \frac{2x}{3} + \frac{y+2x}{2} = 8 - \frac{9y-10}{12} + \frac{3x+7}{4}, \\ \frac{y-3x}{6} = \frac{25}{6} - 2x. \end{cases}$$

$$99. \begin{cases} \frac{2}{x} + \frac{3}{y} + \frac{1}{4} = \frac{18}{y} - \frac{5}{x}, \\ \frac{2}{x} - \frac{1}{y} = \frac{1}{5} \left(\frac{1}{x} + \frac{1}{y} \right) + \frac{1}{12}. \end{cases}$$

$$100. \begin{cases} \frac{1-3x}{7} + \frac{3y-1}{5} = 2, \\ \frac{3x+y}{11} + y = 9. \end{cases}$$

$$101. \begin{cases} 2(2x+3y) = 3(2x-3y) + 10, \\ 4x-3y = 4(6y-2x) + 3. \end{cases}$$

$$102. \left\{ \begin{array}{l} \frac{1}{x} + \frac{2}{y} = a \\ \frac{3}{x} + \frac{4}{y} = b \end{array} \right\}. \quad 103. \left\{ \begin{array}{l} \frac{a}{x} + \frac{b}{y} = c, \\ \frac{b}{x} + \frac{a}{y} = d. \end{array} \right.$$

$$104. \begin{cases} \frac{7}{x} + \frac{5}{y} = 19, \\ \frac{8}{x} - \frac{3}{y} = 7. \end{cases}$$

$$105. \begin{cases} \frac{m}{nx} + \frac{n}{my} = m + n, \\ \frac{n}{x} + \frac{m}{y} = m^2 + n^2. \end{cases}$$

$$106. \begin{cases} \frac{7x}{6} + \frac{5y}{3} = 34, \\ \frac{7x}{8} + \frac{3y}{4} = \frac{5y}{8} + 12. \end{cases}$$

$$107. \begin{cases} \frac{x+y}{8} + \frac{x-y}{6} = 5, \\ \frac{x+y}{4} - \frac{x-y}{3} = 10. \end{cases}$$

$$108. \begin{cases} y + 1 : z :: 5 : 3, \\ \frac{2y}{3} - \frac{5-z}{2} = \frac{41}{12} - \frac{2y-1}{4}. \end{cases}$$

$$109. \begin{cases} b : c :: x : y, \\ x^3 - y^3 = d. \end{cases}$$

$$110. \begin{cases} \frac{y-2}{5} - \frac{10-y}{3} = \frac{z-10}{4}, \\ \frac{2z+4}{3} - \frac{2y+z}{8} = \frac{y+13}{4}. \end{cases}$$

$$111. \begin{cases} x = 4y, \\ \frac{1}{5}(2x + 7y) - 1 = \frac{2}{3}(2x - 6y + 1). \end{cases}$$

$$112. \begin{cases} x + \frac{1}{2}(3x - y - 1) = \frac{1}{4} + \frac{3}{4}(y - 1), \\ \frac{1}{5}(4x + 3y) = \frac{7y}{10} + 2. \end{cases}$$

$$113. \begin{cases} ax + by = c, \\ mx - ny = d. \end{cases}$$

$$114. \begin{cases} \frac{a}{b+y} = \frac{b}{3a+x}, \\ ax + 2by = d. \end{cases}$$

$$115. \begin{cases} b c x = c y - 2 b, \\ b^2 y + \frac{a(c^3 - b^3)}{b c} = \frac{2 b^3}{c} + c^3 x. \end{cases}$$

$$116. \begin{cases} \frac{3x}{10} - \frac{y}{15} - \frac{4}{9} = \frac{x}{12} - \frac{y}{18}, \\ 2x - 2\frac{2}{3} = \frac{x}{12} - \frac{y}{15} + 1\frac{1}{10}. \end{cases}$$

$$117. \begin{cases} 11x + y + 11 = 59 - \frac{2y + 9x}{2} + \frac{3x}{2}, \\ 11 - \frac{7x + 13y}{3} = y - x - \frac{8x - 3y}{4} - \left(x + y + \frac{1}{2}\right). \end{cases}$$

$$118. \begin{cases} \frac{x}{a} - \frac{y}{c} = p, \\ \frac{c}{a - z} + \frac{a}{c + y} = 0. \end{cases}$$

$$119. \begin{cases} \frac{x - 6}{7y} + \frac{4x + 7}{24} - \frac{\frac{1}{7}(7x - y)}{6} = \frac{19 + y}{42} - \frac{\frac{1}{3}(11x + 18)}{56y}, \\ \frac{12x - 15y + \frac{13}{4}}{10y - 8x + \frac{86}{8}} = \frac{93 - 9x}{6x - \frac{14}{5}}. \end{cases}$$

$$120. \begin{cases} 3x + 5y = \frac{(8a - 2b)ab}{a^2 - b^2}, \\ a^2 x - \frac{ab^2 c}{a + b} + (a + b + c)by = b^2 x + (a + 2b)ab. \end{cases}$$

$$121. \begin{cases} \frac{\frac{1}{3}(45x + 4y)}{33} + 2 = y + 1 - \frac{1}{3}(3y + x - 3), \\ \frac{3x + 2y}{6} - \frac{y - 5}{4} = \frac{11x + 152}{12} - \frac{3y + 1}{2}, \end{cases}$$

$$122. \begin{cases} \frac{4x - 3y - 7}{5} = \frac{3x}{10} - \frac{2y}{15} - \frac{5}{6}, \\ \frac{y - 1}{3} + \frac{x}{2} - \frac{3y}{20} - 1 = \frac{y - x}{15} + \frac{x}{6} + \frac{1}{10}. \end{cases}$$

$$123. \begin{cases} \frac{2x-y}{3} + 6 = \frac{2y-x}{2} + \frac{9}{2}, \\ \frac{3x+y}{5} + 1 = \frac{3y+x+13}{10}. \end{cases}$$

$$124. \begin{cases} \frac{1}{7}(x+2) + \frac{1}{4}(y-x) = 2x-8, \\ \frac{1}{8}(2y-3x) + \frac{1}{5}(8x+6y-4) = 3x+4. \end{cases}$$

$$125. \begin{cases} \frac{1}{8}(3x-7y) = \frac{1}{5}(2x+y+1), \\ 8 - \frac{1}{3}(x-y) = 6. \end{cases}$$

$$126. \begin{cases} \frac{2x+y}{9} + \frac{7y+6x+11}{18} = 9\frac{1}{2} - \frac{5x-17}{6}, \\ \frac{1}{3}(5x+3y+2) = \frac{1}{2}(9y+6). \end{cases}$$

$$127. \begin{cases} \frac{\frac{2x}{3} - \frac{5y}{12}}{\frac{7}{4}} - \frac{\frac{3x}{2} - \frac{y}{3}}{\frac{23}{2}} = 2, \\ \frac{x-y}{x+y} = \frac{1}{5}. \end{cases}$$

SIMULTANEOUS EQUATIONS: MORE THAN TWO UNKNOWN QUANTITIES.

Find the values of the unknown quantities:

$$128. \begin{cases} 5x-6y+4z=15, \\ 7x+4y-3z=19, \\ 2x+y+6z=46. \end{cases} \quad 129. \begin{cases} \frac{2}{x} - \frac{5}{3y} + \frac{1}{z} = \frac{85}{27}, \\ \frac{1}{4x} + \frac{1}{y} + \frac{2}{z} = \frac{443}{72}, \\ \frac{5}{6x} - \frac{1}{y} + \frac{4}{z} = \frac{433}{36}. \end{cases}$$

$$130. \begin{cases} x-9y+3z-10u=21, \\ 2x+7y-z-u=683, \\ 3x+y+5z+2u=195, \\ 4x-6y-2z-9u=516. \end{cases}$$

131.
$$\begin{cases} \frac{x}{3} + \frac{y}{5} + \frac{2z}{7} = 58, \\ \frac{5x}{4} + \frac{y}{6} + \frac{z}{3} = 76, \\ \frac{x}{2} + \frac{3z}{8} + \frac{u}{5} = 79, \\ y + z + u = 248. \end{cases}$$
132.
$$\begin{cases} ay + bx = c, \\ cx + az = b, \\ bz + cy = a. \end{cases}$$
133.
$$\begin{cases} \frac{a}{z} + \frac{b}{y} = 1, \\ \frac{b}{y} + \frac{z-c}{x} = 0, \\ x + y + z = 2c. \end{cases}$$
134.
$$\begin{cases} x + y + z = 0, \\ (b+c)x + (c+a)y + (a+b)z = 0, \\ bcx + cay + abz = 1. \end{cases}$$
135.
$$\begin{cases} ax + by + cz = A, \\ a^2x + b^2y + c^2z = A^2, \\ a^3x + b^3y + c^3z = A^3. \end{cases}$$
136.
$$\begin{cases} a(yz - xz - xy) = xyz, \\ b(xz - xy - yz) = xyz, \\ c(xy - yz - xz) = xyz. \end{cases}$$
137.
$$\begin{cases} x + y + z = a + b + c, \\ bx + cy + az = cx + ay + bz = a^2 + b^2 + c^2. \end{cases}$$
138.
$$\begin{cases} \frac{x}{2} + \frac{y}{3} + z = 13, \\ 2x - 3y + \frac{z}{2} = 4, \\ x + 2y : 2y + z :: 5 : 8. \end{cases}$$
139.
$$\begin{cases} 2x + y - 2z = 40, \\ 4y - x + 3z = 35, \\ 3u + t = 13, \\ y + u + t = 15, \\ 3x - y + 3t - u = 49. \end{cases}$$
141.
$$\begin{cases} x + y = xy, \\ x + z = 2xz, \\ 2(y + z) = 3yz. \end{cases}$$
142.
$$\begin{cases} x + 3y + 2z = b, \\ 3x + 5y - 2z = m, \\ 4x - y + z = n. \end{cases}$$
140.
$$\begin{cases} x + 100 = y + z, \\ y + 100 = 2x + 2z, \\ z + 100 = 3x + 3y. \end{cases}$$
143.
$$\begin{cases} ax + by = c, \\ bx + cz = a, \\ cy + az = b. \end{cases}$$

$$144. \begin{cases} x + a(y + z) = m, \\ y + a(x + z) = n, \\ z + a(x + y) = p. \end{cases}$$

$$145. \begin{cases} 2x - \frac{3}{4}y = 93 - \frac{1}{2}x - \frac{1}{4}y, \\ 7x - 5z = x + y - 86, \\ \frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z = 58. \end{cases}$$

$$146. \begin{cases} 3x - 100 = 5y + 360, \\ 2\frac{1}{2}x + 200 = 16\frac{1}{2}z - 610, \\ 2y + 3z = 548. \end{cases}$$

$$147. \begin{cases} \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \\ \frac{x}{a} + \frac{y}{c} + \frac{z}{b} = 1, \\ \frac{x}{b} + \frac{y}{a} + \frac{z}{c} = 1. \end{cases}$$

$$148. \begin{cases} \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 3, \\ \frac{a}{x} + \frac{b}{y} - \frac{c}{z} = 1, \\ \frac{2a}{x} - \frac{b}{y} - \frac{c}{z} = 0. \end{cases}$$

$$149. \begin{cases} v + x + y + z = 14, \\ 2v + x = 2y + z - 2, \\ 3v - x + 2y + 2z = 19, \\ \frac{v}{3} + \frac{x}{4} + \frac{y}{5} + \frac{z}{2} = 4. \end{cases}$$

PROBLEMS INVOLVING TWO OR MORE UNKNOWN QUANTITIES.

150. A and B start together from the foot of a mountain to go to the summit. A should reach the summit half an hour before B, but missing his way goes a mile and back again needlessly, during which he walks at twice his former pace; he reaches the top six minutes before B. C starts twenty minutes after A and B, and walking at the rate of two and one-seventh miles per hour arrives at the summit ten minutes after B. Find the rates of walking of A and B, and the distance from the foot to the summit of the mountain.

151. A crew which can pull at the rate of 9 miles an hour finds that it takes twice as long to come up a river as to go down. At what rate does the river flow?

152. The quantity of water which flows from an orifice is proportional to the product of the area of the orifice and the velocity of the water. There are two orifices in a reservoir, the areas being as 5 : 13, and the velocities are as 8 : 7, and from one there issues in a certain time 561 cubic feet more than from the other. How much water does each orifice discharge in this time?

153. A certain number of persons were divided into three classes, so that the majority of the first and second together over the third was 10 less than four times the majority of the second and third together over the first; but if the first had 30 more, and the second and third together 29 less, the first would have outnumbered the last two by one. Find the number in each class when the whole number was 34 more than eight times the majority of the third over the second.

154. Two trains, 92 feet long and 84 feet long respectively, are moving with uniform velocities on parallel rails; when they move in opposite directions, they are observed to pass each other in one second and a half; but when they move in the same direction, the faster train is observed to pass the other in six seconds. Find the rate at which each train moves.

155. A railroad runs from A to C. A freight-train starts from A at 12 o'clock and a passenger-train at 1 o'clock. After going two-thirds of the distance the freight-train breaks down, and can travel at only three-fourths of its former rate. The rate of the passenger-train is double the diminished rate of the freight-train. At 40 minutes past 2 o'clock a collision of the two trains occurs 10 miles from C. Find the distance from A to C and the rates of the trains.

156. A stage set out from C to G with a certain number of

passengers, four more being on the outside than inside. The fare of seven outside passengers was half a dollar less than that of four inside passengers, and the whole fare received at the outset amounted to \$45. At the end of half the journey it took up three more outside and one more inside passenger at proportional rates, in consequence of which the whole fare received was $1\frac{2}{5}$ times what it was before. What was the number of passengers at the start and the fare of each?

157. A and B are two towns situated 24 miles apart on the same bank of a river. A man goes from A to B in seven hours by rowing the first half of the distance and walking the second half. In returning he walks the first half at three-fourths of his former rate, but the stream being with him he rows the last half at double his rate in going, and he accomplishes the whole distance in six hours. Find his rates of walking and rowing.

158. If there were no accidents it would take half as long to travel the distance from A to B by railroad as by coach, but three hours being allowed for accidental stoppages by the former, the coach will travel the distance, all but fifteen miles, in the same time; if the distance were two-thirds as great as it is, and the same time allowed for railway stoppages, the coach would take exactly the same time. Required the distance.

159. Five players, A, B, C, D, E, throw dice upon condition that he who has the lowest throw shall give all the others the sum which they already have. Each loses in turn, commencing with A, and at the end of the fifth game each has the same sum, \$32. How much had each at first?

160. A person has three ingots composed of three different metals in different proportions. A pound of the first consists of 7 ounces of silver, 3 of copper, 6 of tin; one pound of the second consists of 12 ounces of silver, 3 of copper, 1 of tin; a pound of the third consists of 4 ounces of silver, 7 of cop-

per, 5 of tin. How much of each of the ingots must be taken to form another of one pound weight consisting of 8 ounces of silver, $3\frac{3}{4}$ of copper, $4\frac{1}{4}$ of tin?

161. A railway-train in running from Boston to New York meets with an accident, which causes it to diminish its speed to $\frac{1}{x}$ th of what it was before, and it is in consequence c hours late. If the accident had happened a miles nearer New York the train would have been b hours late. Find the rate of the train before the accident occurred.

162. A railway-train after travelling one hour meets with an accident which delays it one hour, after which it proceeds at three-fifths of its former rate, and arrives at the terminus 3 hours behind time. Had the accident occurred 50 miles farther on, the train would have arrived 1 hour 20 minutes sooner. Required the length of the trip.

163. A mass of tin and lead weighing 120 lbs. in vacuo loses 14 lbs. when weighed in water, and it is known that 37 lbs. of tin loses 5 lbs. and 23 lbs. of lead loses 2 lbs. in water. What are the respective weights of tin and lead?

164. A offers to run three times around a course while B runs twice around, but he gets only 150 yards of his third round finished when B wins. He then offers to run four time around for B's thrice, and now quickens his pace in the ratio of 4 to 3. B also quickens his in the ratio of 9 to 8, but in the second round falls off to his original pace in the first race, and in the third round goes only 9 yards for 10 he went in the first race, and accordingly this time A wins by 180 yards. Determine the length of the course.

INVOLUTION.

Expand the following expressions :

165. $\{(a^2)^3\}^4$.

179. $\left(\frac{3a+6b}{4a^2}\right)^3$.

166. $\left\{\left(\frac{a^3b^2}{c^2}\right)^2\right\}^3$.

180. $\left\{\frac{2a^2x(x-y)}{3y^4z^3}\right\}^8$.

167. $(6a^{-2}b^{-1})^3$.

181. $\left(x + \frac{1}{x}\right)^2$.

168. $(11a^{-1}b^2)^2$.

182. $(5-4x)^4$.

169. $(2a^{-1}x^{-2})^5$.

183. $(5a-10x)^3$.

170. $(2+x)^3$.

184. $\left\{\frac{a^2x^2+2ax^3}{2ab-2bc}\right\}^3$.

171. $(1+x)^4$.

185. $\left\{\frac{x+\frac{p}{2}}{\frac{x}{2}+\frac{y}{4}}\right\}^2$.

172. $(3-2x)^3$.

173. $(a-b)^3(a+b)^3$.

174. $(x-2)^4$.

175. $(2x+3)^4$.

176. $(ax+by)^3+(ax-by)^3$.

186. $\left\{\frac{4ac^2-5c^3}{9a^2-4b^2}\right\}^4$.

177. $(ax+by)^4+(ax-by)^4$.

187. $(1-x+x^2+x^3)^2$.

178. $(1+x)^5-(1-x)^5$.

188. $(1+2x+3x^2+4x^3)^2$.

189. $(a+b+c+d)^2-(a-b+c-d)^2$.

190. $(a+b+c+d)^2+(a-b+c-d)^2$.

191. $(1+3x+3x^2+x^3)^2$.

193. $(2a^2+2x+x^2)^3$.

192. $(1-6x+12x^2-8x^3)^2$.

194. $(5a-x^2-bx)^2$.

195. $(ax+ac-2cx)^2$.

196. $(a^3+3a^2b+3ab^2+b^3)^2$.

197. $(4a+2b-b^2)^2$.

$$198. (7a^2 - 4ax + 3x^2)^2.$$

$$199. (a^2c^2 - 2abcx + b^2x^2)^2.$$

$$200. (2a^2 - ax + x^2)^3.$$

$$207. \left(\frac{2x}{3} + \frac{1}{x} - \frac{x^2}{4}\right)^3.$$

$$201. (a^2 - 2ax + x^2)^3.$$

$$202. (10a + 3ax - x^2)^3.$$

$$208. \left(\frac{a}{3} - 4 - \frac{a^2}{3x}\right)^3.$$

$$203. (1 - 2x + 3x^2)^3.$$

$$204. (x^3 - x^2 + x)^3.$$

$$209. \left(\frac{3ax}{2} - \frac{a}{x} + \frac{2x^2}{3a^2}\right)^3.$$

$$205. (a + 2bc - c)^3.$$

$$206. \left(x + 1 - \frac{1}{x}\right)^3.$$

$$210. \left(\frac{b}{2a} - \frac{3a^2}{b^2} - \frac{2a}{b}\right)^3.$$

EVOLUTION: SQUARE ROOT.

Extract the square roots as indicated:

$$211. \sqrt{5x^2 + 1 - 6x + 12x^3 + 4x^4}.$$

$$212. \sqrt{x^4 - 2x^2y^2 - 2x^2 + y^4 + 2y^2 + 1}.$$

$$213. \sqrt{x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6}.$$

$$214. \sqrt{1 - 2y + 7y^2 - 2y^3 + 5y^4 + 12y^5 + 4y^6}.$$

$$215. \sqrt{4a^4 + 12a^3x + 13a^2x^2 + 6ax^3 + x^4}.$$

$$216. \sqrt{9 - 24x + 58x^2 - 116x^3 + 129x^4 - 140x^5 + 100x^6}.$$

$$217. \sqrt{16a^4 - 40a^3b + 25a^2b^2 - 80ab^2x + 64b^2x^2 + 64a^2bx}.$$

$$218. \sqrt{4x^2y^4 - 12x^3y^3 + 17x^4y^2 - 12x^5y + 4x^6}.$$

$$219. \sqrt{25x^4y^2 - 30x^3y^3 + 29x^2y^4 - 12xy^5 + 4y^6}.$$

$$220. \sqrt{16x^4 - 24x^3y + 25x^2y^2 - 12xy^3 + 4y^4}.$$

$$221. \sqrt[3]{(x^4 + 19x^2 + 25 - 6x^3 - 30x)}.$$

$$222. \sqrt[3]{(25x^2 - 20xy + 4y^2 + 9z^2 - 12yz + 30xz)}.$$

$$223. \sqrt[3]{(1 + 2\frac{1}{8}x^2 - \frac{1}{2}x^5 + \frac{1}{16}x^6 - \frac{3}{8}x - \frac{7}{8}x^3 + \frac{7}{8}x^4)}.$$

$$224. \sqrt{\left(\left(\frac{x}{y}\right)^2 - xy + \frac{1}{4}x^4 - 2 + \frac{x^3}{y} + \frac{y^2}{x^2}\right)}.$$

$$225. \sqrt[3]{(x^{2m} + 2x^{m+n} - 2x^{m+1} + x^{2n} - 2x^{n+1} + x^2)}.$$

$$226. \sqrt{\left(m^2 + 2m - 1 - \frac{2}{m} + \frac{1}{m^2}\right)}.$$

$$227. \sqrt{\left(\frac{a^2}{x^2} - 2 + \frac{x^2}{a^2} + \frac{2a^2}{x} - 2x + a^2\right)}.$$

$$228. \sqrt{\left(\frac{x^4}{4} + 6ax^2 - 4bx^2 + 36a^2 - 48ab + 16b^2\right)}.$$

$$229. \sqrt[3]{(a^{2m} + 2a^{2m-1} + 3a^{2m-2} + 2a^{2m-3} + a^{2m-4})}.$$

$$230. \sqrt{\left(x^4 - 4x^2 + 12 - \frac{16}{x^2} + \frac{16}{x^4}\right)}.$$

$$231. \sqrt{\left(9y^2 - 30y + 31 - \frac{10}{y} + \frac{1}{y^2}\right)}.$$

$$232. \sqrt[3]{(1 - 4x + 10x^2 - 20x^3 + 25x^4 - 24x^5 + 16x^6)}.$$

$$233. \sqrt{\left(4a^2 - 12ab + ab^2 + 9b^2 - \frac{3b^3}{2} + \frac{b^4}{16}\right)}.$$

$$234. \sqrt{\left(36x^4 - 36x^3 + 17x^2 - 4x + \frac{4}{9}\right)}.$$

$$235. \sqrt{\left(x^4 + 8x^2 + 24 + \frac{16}{x^4} + \frac{32}{x^2}\right)}.$$

$$236. \sqrt[3]{(9a^2 - 6ab + 30ac + 6ad + b^2 - 10bc - 2bd + 25c^2 + 10cd + d^2)}.$$

$$237. \sqrt[3]{(x^4 - 2ax^3 + (a^2 + 2b^2)x^2 - 2ab^2x + b^4)}.$$

$$238. \sqrt[3]{(x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64)}.$$

$$239. \sqrt{\left(\frac{4x^2}{9y^2} - \frac{x}{z} - \frac{16x^2}{15yz} + \frac{9y^2}{16z^2} + \frac{6xy}{5z^2} + \frac{16x^2}{25z^2}\right)}.$$

$$240. \sqrt{\left(9x^2 - 30ax - 3a^2x + 25a^2 + 5a^3 + \frac{a^4}{4}\right)}.$$

$$241. \sqrt{\left(a^{2m}x^{2n} + 10ca^{2m-2}x^{2n+1} - 6a^{m+1}x^{n-1} + 25c^2a^{2m-4}x^{2n+2} - 30ca^{m-1}x^n + \frac{9a^2}{x^2}\right)}.$$

$$242. \sqrt{\left(\frac{9}{4} + 6x - 17x^2 - 28x^3 + 49x^4\right)}.$$

$$243. \sqrt{\left(9x^4 - 3ax^3 + 6bx^3 + \frac{a^2x^2}{4} - abx^2 + b^2x^2\right)}.$$

$$244. \sqrt[3]{\left(\frac{4}{3}a^2x^4 - \frac{4}{3}abx^3z + \frac{8}{3}a^2bx^2z^2 + b^2x^2z^2 - 4ab^2xz^3 + 4a^2b^2z^4\right)}.$$

$$245. \sqrt{\left(\frac{9a^{2m-2}c^2}{4d^6p} - \frac{3a^{m+n-1}b^{2n-1}c}{d^{8p-3}} - \frac{2^8a^{m-1}bx^c}{d^{3p}} + a^{2m}b^{4m-2}d^6 + \frac{2^9}{3}a^nb^{x+2n-1}d^8 + \frac{2^{16}b^{2x}}{9}\right)}.$$

Extract the following roots:

$$246. \sqrt[4]{(16a^4 - 96a^3x + 216a^2x^2 - 216ax^3 + 81x^4)}.$$

$$247. \sqrt[4]{(625 + 2000x + 2400x^2 + 1280x^3 + 256x^4)}.$$

$$248. \sqrt[4]{(1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8)}.$$

$$249. \sqrt[4]{\{x^4 - 2(a+b)x^3 + (a^2 + 4ab + b^2)x^2 - 2ab(a+b)x + a^2b^2\}^2}.$$

$$250. \sqrt[3]{\{x^4 - 2x^3y + 3x^2y^2 - 2xy^3 + y^4\}^4}.$$

EVOLUTION: CUBE ROOT.

Extract the cube roots of the following expressions:

251. $a^6 - 40a^3 + 6a^5 + 96a - 64.$

252. $a^6 - 6a^5 + 15a^4 - 20a^3 + 15a^2 - 6a + 1.$

253. $8x^6 - 36ax^5 + 102a^2x^4 - 171a^3x^3 + 204a^4x^2 - 144a^5x + 64a^6.$

254. $x^6 - 3x^5 + 6x^4 - 7x^3 + 6x^2 - 3x + 1.$

255. $x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1.$

256. $y^6 - 6y^5 + 6y^4 + 16y^3 - 12y^2 - 24y - 8.$

257. $1 - 2x^2$ to four terms.

258. $1 - x - x^2$ to four terms.

259. $a^3 - x^3$ to four terms.

260. $1 - 3x + 2x^2$ to four terms.

261. $1 + 9x + 18x^2 - 27x^3 - 54x^4 + 81x^5 - 27x^6.$

262. $x^3 - 3x^2 + 9x - 13 + \frac{18}{x} - \frac{12}{x^2} + \frac{8}{x^3}.$

263. $27a^3 - 54a^2b + 36ab^2 + 54a^2c - 8b^3 - 72abc + 24b^2c + 36ac^2 - 24bc^2 + 8c^3.$

264. $1 - 9x + 27x^2 - 27x^3 + 6y - 36xy + 54x^2y + 12y^2 - 36xy^2 + 8y^3.$

265. $27z^6 - 54az^5 + 63a^2z^4 - 44a^3z^3 + 21a^4z^2 - 6a^5z + a^6.$

266. $8x^6 + 48cx^5 + 60c^2x^4 - 80c^3x^3 - 90c^4x^2 + 108c^5x - 27c^6.$

267. $1 - 9x + 39x^2 - 99x^3 + 156x^4 - 144x^5 + 64x^6.$

$$268. 1 - 3x + 6x^2 - 10x^3 + 12x^4 - 12x^5 + 10x^6 - 6x^7 + 3x^8 - x^9.$$

$$269. 8 - 12x^{3n-1} + 6x^{6n-2} - x^{9n-3}.$$

$$270. \frac{a^3 y^3}{b^6 c^3} + \frac{3a^2 c y^4}{b^4 d} - \frac{3a^3 y^2}{b^4 c^2} + \frac{3a c^5 y^5}{b^2 d^2} - \frac{6a^2 c^2 y^3}{b^2 d} \\ + \frac{3a^3 y}{b^2 c} + \frac{c^9 y^6}{d^3} - \frac{3a c^6 y^4}{d^2} + \frac{3a^2 c^3 y^2}{d} - a^3.$$

$$271. (a+b)^6 m x^3 + 6a c (a+b)^4 m x^2 + 12a^2 p c^2 (a+b)^2 m x + 8a^3 p c^3.$$

$$272. 27x^6 - 54x^5 + 63x^4 - 44x^3 + 21x^2 - 6x + 1.$$

$$273. x^3 + 6x^2 y + 12xy^2 + 8y^3 - 3x^2 z - 12xyz - 12y^2 z + 3xz^2 + 6yz^2 - z^3.$$

$$274. 8m^3 - 36m^2 n + 54mn^2 - 27n^3 - 12m^2 r + 36mnr - 27n^2 r + 6mr^2 - 9nr^2 - r^3.$$

$$275. m^3 + 3m^2 - 5 + \frac{3}{m^2} - \frac{1}{m^3}.$$

Find the sixth roots of—

$$276. 1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6.$$

$$277. 729x^6 - 1458x^5 + 1215x^4 - 540x^3 + 135x^2 - 18x + 1.$$

$$278. m^6 - 12m^5 + 60m^4 - 160m^3 + 240m^2 - 192m + 64.$$

IV.

REDUCTION OF RADICALS.

Introduce the coefficients under the radical sign :

1. $\frac{2}{3}\sqrt[4]{2}$; $4\sqrt[4]{3}$; $3\sqrt[4]{7}$; $5\sqrt[4]{9}$; $2\sqrt[4]{6}$.

2. $\frac{2x}{3b}\sqrt{\frac{2}{3}}$; $3x\sqrt[3]{b^2c^2}$; $(a+b)\sqrt[4]{24}$.

3. $3\sqrt[3]{(a+b)}$; $4\sqrt[4]{2}$; $2\sqrt[3]{7}$; $6a\sqrt[3]{\frac{m}{3a}}$.

4. $a\sqrt{m}$; $c^2\sqrt[3]{am}$; $\frac{5}{2}\sqrt[3]{9\frac{3}{2}}$.

5. $3\sqrt[3]{\frac{3}{4}}$; $3\sqrt{a}$; $4a\sqrt[3]{3x}$.

6. $2ax\sqrt{\left(\frac{3a}{4x}\right)}$.

7. $(m+n) \cdot \sqrt{\left(\frac{m-n}{m+n}\right)}$.

8. $(a+b) \left(\frac{1}{a^2-b^2}\right)^{\frac{1}{2}}$.

9. $\left(\frac{x-y}{x+y}\right) \cdot \left(\frac{x^2+xy}{x^2-2xy+y^2}\right)^{\frac{1}{2}}$.

10. Express \sqrt{a} ; $\sqrt[3]{a^2}$; $\sqrt[4]{a^5}$; $\sqrt{(ab^3c^2)}$; $\sqrt[3]{(abc)^4}$; $\sqrt[5]{(a^2bc^{10})^3}$; and $\sqrt[n]{(a^m b^r c^s)^t}$ with fractional indices.

11. Express $a^{\frac{1}{2}}$; $b^{\frac{2}{3}}$; $c^{\frac{3}{4}}$; $a^{\frac{1}{2}}b^{\frac{2}{3}}$; $(abc)^{\frac{1}{2}}$; $a^{\frac{2}{3}}b^{\frac{3}{4}}$; $(a^5b^3c)^{\frac{2}{3}}$; $(a^2b^4c^6m^7)^{\frac{2}{3}}$; and $\left(a^{\frac{1}{r}}b^{\frac{2}{n}}c^{\frac{rs}{m}}\right)^{\frac{r}{m}}$ with the radical sign.

12. Express $2^{\frac{3}{2}}$; $7^{\frac{1}{2}}$; $2^{\frac{1}{3}}$; $(1\frac{1}{2})^{\frac{2}{3}}$; $(\frac{3}{4})^{-\frac{2}{3}}$; $3^{\frac{2}{3}}$; $(\sqrt{a^5})^{-\frac{2}{3}}$ as equivalent surds with indices whose numerator is in each case + 1.

13. Reduce a ; 3 ; $4\frac{1}{2}$; $2a$; $3a^2b$; $4x^2y^3$ to equivalent surds having index $\frac{1}{2}$.

14. Reduce the preceding to index $-\frac{1}{3}$.

15. Reduce the preceding to index $\frac{1}{4}$.

16. Reduce a^2 ; $\sqrt{3}$; $2a^2b^3$; ac^2 ; $4\frac{2}{3}$; 3^{-2} ; and $(x^{-1}y^{-2}z^3)^{-1}$ to equivalent surds having index $-\frac{1}{2}$.

17. Reduce the preceding to index $\frac{1}{3}$.

18. Reduce $4\sqrt{3}$; $5\sqrt{5}$; $2\sqrt{31}$; $4\sqrt{a}$; $\frac{3}{4}(\frac{2}{3})^{\frac{1}{2}}$; and $\frac{1}{2}(\frac{a^2}{b})^{-\frac{2}{3}}$ to entire surds.

Reduce to the simplest forms:

$$19. \sqrt[4]{32a^6b^2c^4}.$$

$$20. 4\sqrt[3]{16a^4c^3b}.$$

$$21. 2\sqrt{\frac{3}{16}}.$$

$$22. \sqrt[3]{\frac{11ab^3}{16c^3}}.$$

$$23. \sqrt[3]{16a^5x^4}.$$

$$24. \sqrt{\frac{8a^3x^2y^2}{32c^4y^3}}.$$

$$25. \sqrt{a^5(a^6 - a^4x^2)}.$$

$$26. \sqrt[3]{16y^5(a^3x^4 - x^7)}.$$

$$27. \sqrt[3]{432}.$$

$$28. \sqrt{\frac{3}{5}}.$$

$$29. \frac{2}{3}\left(\frac{ab}{3}\right)^{\frac{1}{2}}.$$

$$30. \frac{a}{b}\left(\frac{3}{4}\right)^{\frac{1}{2}}.$$

$$31. \frac{2}{3}(3\frac{1}{2})^{\frac{1}{2}}.$$

$$32. \frac{4}{5}(\frac{2}{3})^{\frac{2}{3}}.$$

$$33. \frac{3}{4}a(\frac{1}{2}b)^{-\frac{2}{3}}.$$

$$34. 2a(\frac{3}{5}a^2)^{-\frac{2}{3}}.$$

$$35. \frac{2}{5}(\frac{3}{4}m)^{\frac{2}{3}}.$$

$$36. (am - pq)\left(\frac{am + pq}{am - pq}\right)^{\frac{1}{2}}.$$

$$37. \sqrt[3]{135}.$$

$$38. \sqrt{162}.$$

39. $\sqrt[4]{80}$.

40. $7\sqrt[5]{324}$.

41. $\frac{1}{2}\sqrt[3]{\frac{2}{7}}$.

42. $\frac{1}{2}\left(-\frac{11a}{704m^5}\right)^{-\frac{1}{2}}$.

43. $(a^3m^6 - a^6m^3 + a^6m^6)^{\frac{1}{2}}$.

48. $\sqrt{50a^2 - 100ab + 50b^2}$.

49. $\sqrt{63c^4y - 42c^2y^2 + 7y^3}$.

50. $\sqrt[3]{x^4 + 3x^3y + 3x^2y^2 + xy^3}$.

51. $\sqrt[3]{a^4 - 3a^3b + 3a^2b^2 - ab^3}$.

44. $\sqrt{\frac{ab^2}{6(a+x)}}$.

45. $\frac{a}{b}\sqrt{\frac{c^2m^2}{a^2n}}$.

46. $\sqrt[n]{a^{m+n}x}$.

47. $\sqrt[q]{\frac{(az - z^2)^{2q}(b+z)}{c+z}}$.

ADDITION AND SUBTRACTION OF RADICALS.

Perform the operations indicated :

52. $p\sqrt[5]{a} + q\sqrt[5]{a}$.

53. $\sqrt[5]{a^7} + b\sqrt[5]{a^2}$.

54. $\sqrt{(9p+18)} + 4\sqrt{(p+2)}$.

55. $a\sqrt{(b^2c + b^3)} + \sqrt{(c^3 + bc^2)}$.

56. $\sqrt[3]{(16a+24)} + \sqrt[3]{(54a+81)}$.

57. $\sqrt[3]{(a^4b + a^3bc)} + b\sqrt[3]{(ab + bc)}$.

58. $\sqrt{(a^2b^2 - b^2c^2)} - \sqrt{(a^4c^2 - a^2c^4)}$.

59. $\sqrt[3]{(64a+128)} - \sqrt[3]{(8a+16)}$.

60. $\sqrt[3]{(a^2b^3 - b^4c)} - \sqrt[3]{(a^5 - a^3bc)}$.

61. $3\sqrt[4]{(16a^2 - 16b^2)} - \sqrt[4]{(a^2 - b^2)}$.

62. $\sqrt[4]{(a^{11}b^7 - a^9b^5)} - \sqrt[4]{(a^7b^{11} - a^5b^9)}$.

$$63. 5\sqrt{a^8} - (4a)^{\frac{1}{2}} - 3a^{\frac{1}{2}} + \sqrt{16a}.$$

$$64. 2\sqrt[3]{2x} + 6\sqrt[3]{4x^2} + \sqrt[3]{8x^3}.$$

$$65. \sqrt[3]{\frac{1}{4}} + \sqrt[3]{\frac{1}{32}}.$$

$$68. \sqrt[3]{\frac{2}{3}} - \sqrt[3]{\frac{2}{27}}.$$

$$66. 12\sqrt[3]{\frac{1}{4}} + 3\sqrt[3]{\frac{1}{82}}.$$

$$69. \sqrt{80a^4x} - \sqrt{20a^2x^3}.$$

$$67. \frac{1}{2}\sqrt{a^2b} + \frac{1}{8}\sqrt{4bx^4}.$$

$$70. 8\sqrt[3]{a^3b} - 2\sqrt[3]{a^6b}.$$

$$71. \sqrt{12} + \sqrt{27} - \sqrt{3} + \sqrt{48}.$$

$$72. \sqrt[3]{40} - 3\sqrt[3]{320} + 4\sqrt[3]{135}.$$

$$73. \sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}} + 2\sqrt[3]{54} - 4\sqrt[3]{\frac{1}{32}}.$$

$$74. \sqrt{18a^6b^3} + \sqrt{50a^3b^3}. \quad 76. \frac{2}{3}\sqrt[3]{\frac{2}{3}} + \frac{1}{4}\sqrt[3]{\frac{256}{9}}.$$

$$75. 3\sqrt{\frac{2}{3}} + 7\sqrt{\frac{27}{16}}.$$

$$77. \sqrt{27} + 2\sqrt{48} + 3\sqrt{108}.$$

$$78. \sqrt{9000} + 4\sqrt{50} + 12\sqrt{288}.$$

$$79. a\sqrt{a^2x} + b\sqrt{b^2x} + c\sqrt{c^2x}.$$

$$80. 7\sqrt[3]{54} + 3\sqrt[3]{16} + \sqrt[3]{432}. \quad 83. 5\sqrt[3]{16} - 2\sqrt[3]{54}.$$

$$81. \sqrt{96} - \sqrt{54}.$$

$$84. 7\sqrt[3]{81} - 3\sqrt[3]{1029}.$$

$$82. 12\sqrt{72} - 3\sqrt{128}.$$

$$85. 2\sqrt[3]{4} + 5\sqrt[3]{32} - \sqrt[3]{108}.$$

$$86. 2\sqrt{3} + 3\sqrt{1\frac{1}{3}} - \sqrt{5\frac{1}{3}}.$$

$$87. \frac{1}{\sqrt[3]{2}} - \frac{1}{\sqrt[3]{16}}.$$

$$88. \sqrt[3]{128} + \sqrt[3]{686} + \sqrt[3]{16} - 4\sqrt[3]{250}.$$

$$89. 2\sqrt{8a^3} - 7a\sqrt{18a} + 5a\sqrt{72a} - \sqrt{50ab^2}.$$

$$90. 7\sqrt[3]{54} + 3\sqrt[3]{16} + \sqrt[3]{2} - 5\sqrt[3]{128}.$$

$$91. \sqrt[m]{2^m a^{mp+3} b^{mn+5}} + \sqrt[m]{3^m a^{3m-mn+3} b^{m+5}} \\ - \sqrt[m]{a^3 b^5 c^{2m}}.$$

$$92. 4\sqrt{18} + 3\sqrt{32} - \sqrt{2} - 4\sqrt{8} + 5\sqrt{98}.$$

$$93. 8\sqrt{\frac{2}{3}} + \sqrt{60} - \frac{11}{5}\sqrt{15} + \sqrt{\frac{2}{3}}.$$

$$94. \sqrt{28} + \sqrt[3]{81} + 2\sqrt{63} - 2\sqrt[3]{24}.$$

$$95. 3b^2(a^3c)^{\frac{1}{2}} + \frac{2}{c}(a^5c^3)^{\frac{1}{2}} - c^4\left(\frac{ac}{b^2}\right)^{\frac{1}{2}}.$$

$$96. 4\sqrt{24} + 2\sqrt{54} - \sqrt{6} + 3\sqrt{96} - 5\sqrt{150}.$$

$$97. 3\sqrt{\frac{2}{3}} - 2\sqrt{\frac{1}{10}} + \sqrt{\frac{5}{2}}.$$

MULTIPLICATION AND DIVISION OF RADICALS.

Perform the operations indicated:

$$98. 2^{\frac{1}{2}} \times 3^{\frac{1}{2}}.$$

$$101. \sqrt{50} \times \sqrt{200}.$$

$$99. \sqrt{6} \times \sqrt{8}.$$

$$102. \sqrt[3]{3a^2b} \times \sqrt[3]{9ab^2}.$$

$$100. \sqrt{14} \times \sqrt{20}.$$

$$103. \sqrt[3]{12ab} \times \sqrt[3]{8a^2b^3}.$$

$$104. \sqrt{(x^2 + x^3y)} \div \sqrt{(x + 2x^2y + x^3y^2)}.$$

$$105. \sqrt{\frac{3ab}{2c}} \times b\sqrt{\frac{ab}{c}}.$$

$$108. \sqrt[4]{64} \times 2\sqrt{2}.$$

$$109. 3a\sqrt{a^5x} \div 2x\sqrt[3]{a^2x^5}.$$

$$106. 4\sqrt[3]{\frac{1}{18}} \times 3\sqrt{8}.$$

$$110. \sqrt{12} \div \sqrt[3]{24}.$$

$$107. 2\sqrt[4]{12} \times \frac{1}{2}\sqrt[4]{\frac{2}{3}}.$$

$$111. 4\sqrt[5]{32} \div \sqrt[3]{16}.$$

$$112. \{2\sqrt{8} \times 4\sqrt[4]{\frac{1}{4}}\} \div \{4\sqrt[3]{\frac{1}{2}} \times 4\sqrt[6]{4}\}.$$

$$113. 3a\sqrt{(x^2 - y^2)^3} \div 4x\sqrt[3]{(x^2 - y^2)}.$$

$$114. (\sqrt[n]{(a - x)}) (\sqrt[n]{(a + x)}).$$

$$115. 5\frac{3}{4}\sqrt[4]{\frac{1}{185}} \div \frac{2}{3}\sqrt{\frac{1}{5}}.$$

$$117. 4\frac{1}{2}\sqrt{a} \div 2\frac{2}{3}\sqrt[3]{ab}.$$

$$116. 4\frac{5}{7}\sqrt{\frac{2}{3}} \div 2\frac{2}{5}\sqrt{\frac{3}{4}}.$$

$$118. 9\frac{2}{3}a^{\frac{1}{n}} \div 4\frac{3}{11}a^{\frac{1}{m}}.$$

$$119. (\sqrt{20} + \sqrt{12}) \div (\sqrt{5} + \sqrt{3}).$$

$$120. \sqrt[3]{16} \times \sqrt{8}; 4a^{\frac{1}{2}} \times 7a^{\frac{3}{2}}; 2\sqrt{3} \times \sqrt[6]{72}; \sqrt[3]{4} \times 7\sqrt[3]{6} \times \frac{1}{2}\sqrt[3]{5}.$$

121. $\left(\frac{ax}{bc} \sqrt{ax}\right) \left(\frac{by}{cd} \sqrt[3]{by}\right) \left(\frac{c^2 d}{a} \sqrt[4]{cz}\right); (x - \sqrt{xy} + y)$
 $(\sqrt{x} + \sqrt{y}).$
122. $(4\sqrt{3} + 3\sqrt{7})(2\sqrt{2} - 4\sqrt{5}); (2\sqrt{3} + \frac{2}{3}\sqrt{\frac{3}{5}})$
 $(3\sqrt{2\frac{1}{2}} - 4\sqrt{3}).$
123. $3\sqrt{2} \div 4\sqrt{3}; 5\sqrt{7} \div 3\sqrt{8}; 2\sqrt{\frac{2}{3}} \div \sqrt{\frac{2}{5}}; 2\sqrt{2\frac{1}{2}} \div 3\sqrt{3\frac{1}{4}}.$
124. $4\sqrt{\frac{2}{3}} \div 3\sqrt[3]{\frac{3}{2}}; 4\sqrt[3]{ax} \div 3\sqrt{ax}.$
125. $\sqrt[5]{ab^{n-1}c^2} \div \sqrt[5]{\frac{a^8 d^{-1}}{c^{n-1}b^{-2}}}.$
126. $\sqrt{\frac{a^m b}{c^2 d}} \div \sqrt{\frac{a^{m-1} c^3}{d^5}}.$
127. $(\sqrt[m]{a^{2m-n} b^{5m+1} c^{2p}}) \cdot (\sqrt[m]{a^n b^{m-1} c^{m-p}}).$
128. $\left(\sqrt{5} + \sqrt{1\frac{1}{4}} - \frac{1}{\sqrt{5}}\right) \cdot (\sqrt{3}).$
129. $\left(\sqrt[3]{4} - \frac{1}{\sqrt[3]{16}} + \frac{1}{\sqrt[3]{2}}\right) \cdot (\sqrt[3]{4}).$
130. $(1 + \sqrt{3} - \sqrt{2}) \cdot (\sqrt{6} - \sqrt{2}).$
131. $(\sqrt{3} + \sqrt{2}) \cdot \left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}}\right).$
132. $\{\sqrt{3x+1} + \sqrt{2x-1}\} \{\sqrt{3x} - \sqrt{2x-1}\}.$
133. $\{\sqrt{a} + \sqrt{a-x}\} \{\sqrt{x} - \sqrt{a-x}\}.$
134. $(a-b) \div \sqrt[3]{a - \sqrt[3]{b}}.$
135. $\{\sqrt{x} + \sqrt{y} + \sqrt{z}\} \{\sqrt{x} - \sqrt{y} + \sqrt{z}\}.$
136. $\{\sqrt{a} + \sqrt{a-x} + \sqrt{x}\} \{\sqrt{a} - \sqrt{a-x} + \sqrt{x}\}.$
137. $(2\sqrt[3]{2} - 3\sqrt[4]{3} + 7\sqrt[6]{5}) \div 5\sqrt{2}.$

138. $(7 + 2\sqrt{6})(9 - 5\sqrt{6})$.
 139. $(4\sqrt{\frac{7}{3}} + 5\sqrt{\frac{1}{2}})(\sqrt{\frac{7}{3}} + 2\sqrt{\frac{1}{2}})$.
 140. $(\sqrt[3]{a^3} - \sqrt[3]{b^3}) \div (\sqrt[3]{a} - \sqrt[3]{b})$.
 141. $(2\sqrt{8} + 3\sqrt{5} - 7\sqrt{2})(\sqrt{72} - 5\sqrt{20} - 2\sqrt{2})$.
 142. $(2\sqrt{2} + 5\sqrt{15} - \sqrt{3})(2\sqrt{2} - 5\sqrt{15} + \sqrt{3})$.
 143. $(9 + 3\sqrt{3} + 3\sqrt{5} + \sqrt{15})(\sqrt{5} - 2)$.
 144. $(c\sqrt{a} + d\sqrt{b})(c\sqrt{a} - d\sqrt{b})$.
 145. $(\sqrt{2} + 3\sqrt{\frac{1}{2}}) \div (\frac{1}{2}\sqrt{\frac{1}{2}})$.
 146. $(3\sqrt{5} + \sqrt{15} - 3 - \sqrt{3}) \div \frac{1}{3}\sqrt{15}$.
 147. $(\sqrt[3]{x^4} - 4xy + 4y\sqrt[3]{x^2} + 4y^2) \div (\sqrt[3]{x^2} + 2\sqrt{xy} + 2y)$.
 148. $(a - b) \div (\sqrt[4]{a} - \sqrt[4]{b})$.
 149. $(x - 4\sqrt[4]{x^3} + 6\sqrt{x} - 4\sqrt[4]{x} + 1) \div (\sqrt{x} - 2\sqrt[4]{x} + 1)$.
 150. $(\sqrt[4]{x^8} - 2\sqrt[5]{a^{12}x^8} + a^8) \div (\sqrt[4]{x} - 2\sqrt[5]{a^4x} + a)$.
-

INVOLUTION AND EVOLUTION OF RADICALS.

Perform the operations indicated :

- | | |
|-----------------------------------|--|
| 151. $(\frac{2}{3}\sqrt{3})^3$. | 158. $\left(2a\sqrt{\frac{xy^3}{z}}\right)^3$. |
| 152. $(3\sqrt[3]{3})^2$. | 159. $(3\sqrt[3]{(a^2 - x^2)})^4$. |
| 153. $(\frac{1}{8}\sqrt{6})^4$. | 160. $\left(2a\sqrt{\frac{x^3(a-y)}{a(x+y)^3}}\right)^3$. |
| 154. $(3 + 2\sqrt{5})^2$. | 161. $\left(2x\sqrt{\frac{x-y}{2z}}\right)^4$. |
| 155. $(\sqrt{x} + 3\sqrt{y})^3$. | 162. $\left(3\sqrt{\frac{a+x}{a-x}}\right)^3$. |
| 156. $(\sqrt{3} - \sqrt{2})^4$. | |
| 157. $(\sqrt{3} + \sqrt{2})^3$. | |

163. $(\sqrt{5} + \sqrt{3})^4.$

165. $\left(a\sqrt{\frac{c}{b}} - \sqrt{cd}\right)^2.$

164. $\sqrt[3]{a^3 \sqrt[3]{x^2}}.$

166. $\left(a^{\sqrt{2}} - \frac{1}{a^{\sqrt{2}}}\right)^2.$

167. $\left[\sqrt{\left(\frac{x}{4} + 3\right)} - \sqrt{\left(\frac{x}{4} - 3\right)}\right]^2.$

168. $[\sqrt{x+a} - \sqrt{x-a}]^2.$

169. $\sqrt[4]{\left(\frac{a\sqrt{b}}{\sqrt[3]{ab}}\right)^8}.$

171. $[\sqrt{x} + \sqrt{x-4}]^2.$

172. $[\sqrt{x-6} + \sqrt{x}]^2.$

170. $\sqrt[6]{(a^3 b \sqrt[5]{a^3 b c})^6}.$

173. $(\sqrt[3]{x} - \sqrt[3]{y})^3.$

174. $[\sqrt{x+y} - \sqrt{x-y}]^2.$

175. $[\sqrt{x} \cdot \sqrt{x+1} - \sqrt{x-1}]^2.$

176. $[\sqrt{x+1} + \sqrt{x} \cdot \sqrt{x-1}]^2.$

177. $\sqrt[3]{27a^3} \sqrt{\frac{x^8}{y^5}}.$

179. $\sqrt[4]{81} \sqrt[3]{\frac{2}{3}}.$

178. $\sqrt[3]{8a^3 x \sqrt[3]{(a-x)^2}}.$

180. $\sqrt[3]{8x^4 \sqrt{(x^2 - y^2)^3}}.$

181. $(9a^3 x - 12ax\sqrt{xy} + 4x^2 y)^{\frac{1}{2}}.$

182. $\sqrt{x\sqrt[3]{x^2}} - 4x\sqrt[3]{x} + 2x\sqrt[6]{x} + 4x - 4\sqrt[6]{x^5} + \sqrt[3]{x^2}.$

183. $\left(\frac{9x}{y} - 24\sqrt{\frac{x}{y}} + 34 - 24\sqrt{\frac{y}{x}} + \frac{9y}{x}\right)^{\frac{1}{2}}.$

GENERAL THEORY OF EXPONENTS.

Remove literal factors to numerators :

$$184. \frac{2a}{bm}; \frac{2}{a}; \frac{3a}{m}; \frac{m^2}{ac^2}; \frac{3abm}{4m^2c^3}; \frac{2a^{\frac{1}{2}}m^{\frac{1}{2}}}{5c\sqrt{m}}; \frac{3a^{\frac{1}{2}}b\sqrt[3]{cm^3}}{a^2b\sqrt[3]{m}};$$

$$\frac{1}{\sqrt[3]{ab^2cm^{\frac{1}{2}}}}; \frac{a^{\frac{1}{2}}b^{\frac{1}{2}}c^{\frac{1}{2}}}{m^{\frac{1}{2}}b^{\frac{1}{2}}c^{\frac{1}{2}}}.$$

Remove literal factors to denominators :

$$185. \frac{4m}{5\sqrt{a}}; \frac{3\sqrt[3]{ab^2c^4}}{5\sqrt[3]{mn^8}}; \frac{b^2}{c}; \frac{3am}{\sqrt{c}}; \frac{2a}{3x^2y}; ab^2c^{\frac{1}{2}};$$

$$\frac{3axy^2}{2a^3\sqrt{m}}; \frac{4ac}{3mx}.$$

Perform operations indicated and simplify :

$$186. \frac{x^{2n} - a^{2n}}{x^n + a^n}.$$

$$188. \{(a^m)^m - \frac{1}{m}\} \frac{1}{m+1}.$$

$$187. \left(\sqrt[6]{\frac{1}{x^3y^2}}\right)(\sqrt[5]{x^2} \cdot \sqrt[3]{y}). \quad 189. \frac{x^{a+b} \cdot x^{a-b} \cdot x^{c-2a}}{x^{c-a}}.$$

$$190. \{-(x^3)^{\frac{1}{2}}\}^{-\frac{1}{2}} \cdot \{-(-x)^{-3}\}^{\frac{1}{2}}.$$

$$191. [\{(a-m)^{-n}\}^{-p}] \div [\{(a^2m)^{-3p}\}^{2n}].$$

$$192. \{ab^2\sqrt[3]{(ab^3)}\sqrt[5]{(ab^4)}\sqrt[4]{(ab^5)}\}^{\frac{1}{5}}.$$

$$193. \left\{(a+x)\sqrt{\left(\frac{b^3}{(a+x)^{\frac{1}{2}}}\right)}\right\}^{\frac{1}{2}}.$$

$$194. (a^2b^{-\frac{1}{2}}c^{\frac{2}{3}})^{-\frac{1}{2}}.$$

$$195. (4x - 2x^{\frac{1}{2}}y^{-\frac{1}{2}} + 2x^{\frac{1}{2}}z^{\frac{1}{2}} - y^{-1} + y^{-\frac{1}{2}}z^{\frac{1}{2}})(2x^{\frac{1}{2}} + y^{-\frac{1}{2}} - z^{\frac{1}{2}}).$$

$$196. (9x^{-9}y - 4x^{-7}y^{-1}) - (-3x^{-4}y - 2x^{-8}).$$

$$197. (a + a^{\frac{2}{3}}b^{-\frac{1}{2}} - a^{\frac{1}{2}}b^{-\frac{2}{3}} - b^{-1}) \div (a^{\frac{2}{3}} + a^{\frac{1}{2}}b^{-\frac{1}{2}} + a^{\frac{2}{3}}b^{-\frac{1}{2}} + a^{\frac{1}{2}}b^{-\frac{2}{3}} + b^{-\frac{1}{2}}).$$

$$198. (x^{-1} + x^{-\frac{1}{2}} - 1 + x^{\frac{1}{2}} + x) \div (x^{-\frac{1}{2}} + x^{\frac{1}{2}} + 1).$$

$$199. (a^{\frac{3}{2}} - a + a^{\frac{1}{2}} + 1 - a^{-\frac{1}{2}} - a^{-1} + a^{-\frac{3}{2}})^2.$$

$$200. \sqrt[3]{(a^{\frac{3}{2}} + 2a^{\frac{1}{2}} - 1 - 2a^{-\frac{1}{2}} + a^{-\frac{3}{2}})}.$$

$$201. \sqrt[3]{(x^{\frac{4}{3}} - 4x + 10x^{\frac{2}{3}} - 16x^{\frac{1}{3}} + 19 - 16x^{-\frac{1}{3}} + 10x^{-\frac{2}{3}} - 4x^{-1} + x^{-\frac{4}{3}})}.$$

$$202. \sqrt[3]{(x^{-1}y^3 - 3x^{-\frac{1}{2}}y + 3x^{\frac{1}{2}}y^{-1} - xy^{-3})}.$$

$$203. \sqrt[3]{(x^2 - 6x^{\frac{5}{3}}y^{\frac{1}{3}} + 21x^{\frac{4}{3}}y^{\frac{2}{3}} - 44xy^{\frac{1}{2}} + 63x^{\frac{2}{3}}y^{\frac{2}{3}} - 54x^{\frac{1}{3}}y^{\frac{5}{3}} + 27y)}.$$

$$204. \left(\sqrt[3]{\left\{ \sqrt{\left(a^{-3} \times \frac{1}{\sqrt{c}} \right) ac} \right\}} \right)^{12}$$

$$205. \left\{ \frac{\sqrt[4]{(a^3 \sqrt[3]{b})}}{\{(a^3)^{\frac{1}{2}} b^{\frac{3}{2}} c^{\frac{3}{2}}\}^{-\frac{1}{4}}} \times \frac{a^3 b^3 c^3}{\{(a^4)^{\frac{1}{2}} b^6\}^{\frac{1}{2}} c^6} \right\}^{-\frac{1}{4}} \\ \times \{a^{-1} \sqrt[4]{b^{-2}} \sqrt[4]{c^{-21}}\}^{\frac{1}{4}}$$

$$206. \frac{\{\sqrt[3]{(x^n)} \sqrt[3]{(x^m)} \sqrt[3]{(x^p)}\}^{rst}}{\{\sqrt[3]{(y^n)} \sqrt[3]{(y)^r} \sqrt[3]{(y^7)}\}^{4sq}} \div \left\{ \left(\frac{x^{mt}}{y^{4s}} \right)^r \right\}^q.$$

$$207. (x^3 y^{-3} - x^{-3} y^3 - 3xy^{-1} + 3x^{-1}y) \div (xy^{-1} - x^{-1}y).$$

$$208. \sqrt[13]{\left(a^3 b^2 \sqrt[3]{\left\{ abc^4 \sqrt[4]{\frac{1}{ab^2 c^3}} \right\}} \right)^{12}}.$$

$$209. (a^3 - a^{\frac{4}{3}} + 2a^{\frac{1}{3}} - 2 - a^{-\frac{2}{3}} + a^{-3}) \div (a^{\frac{2}{3}} + a^{\frac{3}{2}} - a^{-\frac{1}{2}} - a^{-\frac{3}{2}}).$$

$$210. \sqrt[3]{(4x^{\frac{3}{2}} - 12x^{\frac{1}{2}} + 25 - 24x^{-\frac{1}{2}} + 16x^{-\frac{3}{2}})}.$$

$$211. \left(\frac{y}{\sqrt{x}} + \sqrt[4]{xy} - \frac{x}{2\sqrt{y}} \right)^2.$$

$$212. \{(a+b)^{\frac{1}{2}} - (a-b)^{\frac{1}{2}}\}^3.$$

In the first three of the following, factor the results so that one factor shall be free from fractional exponents:

$$213. \left(5\sqrt[4]{a^7} - \frac{6ab}{\sqrt[4]{a}} \right) \left(\sqrt[3]{a} - \frac{7b}{\sqrt[3]{a^2}} \right).$$

$$214. \left(\sqrt[5]{a} b^3 + 3\sqrt[5]{\frac{b^8}{a^4}} \right) \left(\sqrt[3]{ab} + \frac{2}{b^2} \sqrt[3]{\frac{b}{a}} \right).$$

$$215. \left(\sqrt[6]{\frac{1}{a^3 b^2}} - \frac{2\sqrt[3]{b^2 c^3}}{a\sqrt[3]{a}} \right) \left(\sqrt[5]{a^2} - \frac{b}{\sqrt[5]{a^3}} \right).$$

$$216. \left(\frac{b}{c} \sqrt[5]{\frac{ad}{f}} - cd \sqrt[3]{\frac{ac}{bg}} \right) \left(\sqrt[5]{\frac{ab^5 d}{c^5 f}} + \sqrt[3]{\frac{ac^4 d^3}{bg}} \right).$$

$$217. \left(\sqrt[12]{a^{10} b^9} - c \sqrt[10]{a^7} \sqrt[6]{b^5} - \frac{3}{2} a \sqrt[4]{b^3} + \frac{3abc^{30}}{2} \sqrt[10]{\frac{1}{a^4 b^5}} \right) \\ + \left(\sqrt[3]{ab} - \frac{3}{2} \sqrt[6]{a^4 b^3} \right).$$

$$218. (5a^2 - 41ab + 42b^2) \sqrt[12]{a} \div \left(\sqrt[3]{a} - \frac{7b}{\sqrt[3]{a^2}} \right).$$

$$219. \left(\frac{c^2 d}{(a+b)^{\frac{3}{2}}} \right)^{-\frac{1}{2}}.$$

$$220. (x + x^{\frac{1}{2}} + x^{-\frac{1}{2}}) (x^{\frac{1}{2}} + x^{-\frac{1}{2}} - x^{-1}).$$

$$221. (a^n + 1 + a^{-n}) \left(a^{\frac{n}{2}} - a^{-\frac{n}{2}} \right).$$

$$222. \left(x^{\frac{p}{q}} - x^{\frac{r}{s}} \right) \left(x^{\frac{m}{n}} + x^{\frac{t}{u}} \right).$$

$$223. (3x^{\frac{1}{2}} + 2x^{\frac{1}{4}} + 1)^3.$$

$$224. (a^{\frac{1}{2}} + b^{\frac{1}{2}} - c^{\frac{1}{2}})^3.$$

$$225. \frac{x^{\frac{3n}{2}} - x^{-\frac{3n}{2}}}{x^{\frac{n}{2}} - x^{-\frac{n}{2}}}.$$

QUADRATIC SURDS.

Find the square roots of the following Binomial Surds :

226. $36 \pm 10\sqrt{11}$.

233. $9 + \sqrt{56}$.

227. $\frac{3}{4} \pm \sqrt{2}$.

234. $3 - \frac{1}{2}\sqrt{14}$.

228. $41 \pm 24\sqrt{2}$.

235. $a - 2\sqrt{a-1}$.

229. $31 + 10\sqrt{6}$.

236. $1 + 4\sqrt{-3}$.

230. $2a + 2\sqrt{a^2 - b^2}$.

237. $3 \pm 4\sqrt{-1}$.

231. $4 + \sqrt{15}$.

238. $-1 + \sqrt{-8}$.

232. $16 - 6\sqrt{7}$.

239. $a^2 + 2x\sqrt{a^2 - x^2}$.

240. $ab + c^2 + \sqrt{(a^2 - c^2)(b^2 - c^2)}$.

241. $-9 + 6\sqrt{3}$.

247. $3\sqrt{5} + \sqrt{40}$.

242. $1 + (1 - c^2)^{-\frac{1}{2}}$.

248. $3\sqrt{6} + 2\sqrt{12}$.

243. $94 - 42\sqrt{5}$.

249. $\sqrt{18} - 4$.

244. $38 - 12\sqrt{10}$.

250. $\frac{a^2}{4} + \frac{1}{2}b\sqrt{a^2 - b^2}$.

245. $14 - 4\sqrt{6}$.

251. $2\frac{1}{4} - \sqrt{5}$.

246. $\sqrt{32} - \sqrt{24}$.

252. $\sqrt{27} + 2\sqrt{6}$.

RATIONALIZATION.

Reduce the expressions to fractions that shall have rational denominators :

253. $\frac{3ay^4}{\sqrt[3]{2ax^4}}$.

254. $\frac{2a}{\sqrt[n]{x^m}}$.

255. $\frac{1}{\sqrt[3]{x^5}}$.

$$256. \frac{1}{\sqrt{x^{2m}y^{3n}}}.$$

$$266. \frac{a + \sqrt{(a^2 - x^2)}}{a - \sqrt{(a^2 - x^2)}}.$$

$$257. \frac{3}{2\sqrt{x^{2m}}}.$$

$$267. \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}.$$

$$258. \frac{1}{\sqrt[3]{a^4x^2}}.$$

$$268. \frac{2\sqrt{5} + \sqrt{3}}{3\sqrt{5} + 2\sqrt{3}}.$$

$$259. \frac{2a}{3\sqrt[7]{x^m y^n}}.$$

$$269. \frac{2\sqrt{3} + 3\sqrt{2}}{3\sqrt{3} - 2\sqrt{5}}.$$

$$260. \frac{4}{2\sqrt[3]{12}}.$$

$$270. \frac{2\sqrt{3} + \sqrt{11}}{7\sqrt{8} - 8\sqrt{7}}.$$

$$261. \frac{5\sqrt{2} - \sqrt{7}}{3\sqrt{5} + \sqrt{6}}.$$

$$271. \frac{3\sqrt{5} - 2\sqrt{2}}{2\sqrt{5} - \sqrt{18}}.$$

$$262. \frac{1}{\sqrt{5} - \sqrt{2} + 3\sqrt{3}}.$$

$$272. \frac{a\sqrt{m} - m\sqrt{a}}{a\sqrt{m} + m\sqrt{a}}.$$

$$263. \frac{\sqrt{(a+x)} + \sqrt{(a-x)}}{\sqrt{(a+x)} - \sqrt{(a-x)}}.$$

$$273. \frac{2 + 3\sqrt{\frac{2}{3}}}{\frac{1}{3}\sqrt{\frac{1}{2}} - \frac{2}{3}\sqrt{\frac{3}{5}}}.$$

$$264. \frac{\sqrt{(m^2+1)} - \sqrt{(m^2-1)}}{\sqrt{(m^2+1)} + \sqrt{(m^2-1)}}.$$

$$274. \frac{4}{\sqrt{3} + \sqrt{2} + 1}.$$

$$265. \frac{a + \sqrt{(a^2 - 1)}}{a - \sqrt{(a^2 - 1)}}.$$

$$275. \frac{1}{\sqrt{5} - \sqrt[4]{2}}.$$

$$276. \frac{15}{\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80}}.$$

$$277. \frac{3}{\sqrt{3} + 2\sqrt{5} - \sqrt{2}}.$$

$$278. \frac{\sqrt[3]{4} - 8\sqrt[3]{72}}{\sqrt[3]{2} - 4\sqrt[3]{3}}.$$

$$279. \frac{10}{\sqrt[3]{7} - \sqrt[3]{5}}.$$

$$280. \frac{\sqrt[3]{3}}{\sqrt[3]{9} + \sqrt[3]{10}}.$$

$$281. \frac{4}{\sqrt[4]{4} + \sqrt[4]{5}}.$$

SIMPLIFICATION OF COMPLEX RADICAL FORMS.

Simplify:

282. $(a^{\frac{1}{2}} + b^{\frac{1}{2}})^3 (a^{\frac{1}{2}} - b^{\frac{1}{2}}).$

285. $\frac{\sqrt{2} + 1}{\sqrt{2} - 1} + \frac{\sqrt{2} - 1}{\sqrt{2} + 1}.$

283. $\frac{\sqrt{a^2 x - 2 a x^2 + x^3}}{\sqrt{a^2 + 2 a x + x^2}}.$

286. $\frac{\sqrt{12}}{(1 + \sqrt{2})(\sqrt{6} - \sqrt{3})}.$

284. $x \sqrt[3]{\left(\frac{8 a^4}{27 b^3} + \frac{16 a^3}{27 b^2}\right)}.$

287. $\frac{2 + \sqrt{3}}{2 - \sqrt{3}} - \frac{2 - \sqrt{3}}{2 + \sqrt{3}}.$

288. $\frac{1}{x - \sqrt{x^2 - 1}} + \frac{1}{x + \sqrt{x^2 - 1}}.$

289. $\left(\frac{\sqrt{a^2 - x^2}}{2} + \frac{x}{2}\right)^2.$

290. $\left\{\sqrt{\frac{x + \sqrt{x^2 - a}}{2}} + \sqrt{\frac{x - \sqrt{x^2 - a}}{2}}\right\}^2.$

291. $\frac{1 + \frac{\sqrt{a^2 - x^2}}{\sqrt{a^2 + x^2}}}{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}.$

292. $\frac{\sqrt{1 - x^2} + \frac{x^2}{\sqrt{1 - x^2}}}{1 - x^2}.$

293. $\frac{ax}{\sqrt{a + x}} - \frac{2ax^2}{(a + x)^{\frac{3}{2}}} + \frac{ax^3}{(a + x)^{\frac{5}{2}}}.$

294. $\frac{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}}{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}} + \frac{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}}{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}}.$

295. $\frac{\sqrt{1 - x} + \frac{1}{\sqrt{1 + x}}}{1 + \frac{1}{\sqrt{1 - x^2}}}$

296. $\frac{x - 1}{x + 1} \left\{ \frac{x - 1}{\sqrt{x - 1}} + \frac{1 - x}{x + \sqrt{x}} \right\}.$

$$297. \frac{6c^2}{x-1} \sqrt{\left(\frac{4x^3 - 8x^2 + 4x}{3c^3} \right)}.$$

$$298. \frac{x}{x-7} \{ \sqrt[3]{(3p^2x^3 - 63p^2x^2 + 441p^2x - 1029p^2)} \}^6.$$

$$299. 2(n-1) \sqrt[3]{\left(-\frac{1}{2n^4 - 6n^3 + 6n^2 - 2n} \right)}.$$

$$300. 2(n-1) \sqrt[3]{63} + \frac{1}{3} \sqrt[3]{112} - \frac{\sqrt[3]{28n^4}}{n^2} \\ + \sqrt[3]{175(n-1)^2c^2} \times \frac{2}{3c} - 2 \sqrt{\left(\frac{7n^2}{36} \right)}.$$

$$301. \frac{\sqrt{x} + \sqrt{y}}{3\sqrt{y}} - \frac{\sqrt{x} - \sqrt{y}}{3\sqrt{x}}.$$

$$302. \sqrt{\{(x + x^{-1})^2 - 4(x - x^{-1})\}}.$$

$$303. \sqrt{(x^3 + 2x^2y + xy^2)} + \sqrt{(x^3 - 2x^2y + xy^2)}.$$

$$304. \frac{a-b}{\sqrt{a}-\sqrt{b}} - \frac{a+b}{\sqrt{a}+\sqrt{b}}.$$

$$305. \left\{ \frac{1}{x^2-a^2} - \frac{1}{x^2+a^2} - \frac{\frac{1}{x+a} - \frac{1}{x-a}}{\frac{x^2+a^2}{a}} \right\}^{\frac{1}{2}}.$$

$$306. \frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}} - \frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}.$$

$$307. \left(\frac{2-\sqrt{3}}{2+\sqrt{3}} \right)^{\frac{1}{2}}.$$

$$309. \frac{1}{\sqrt{(7-4\sqrt{3})}}.$$

$$308. \frac{1}{\sqrt{(5+\sqrt{24})}}.$$

$$310. \frac{\sqrt{(12+6\sqrt{3})}}{1+\sqrt{3}}.$$

$$311. \sqrt{(3+\sqrt{5})} + \sqrt{(3-\sqrt{5})}.$$

$$312. \frac{(3+\sqrt{3})(3+\sqrt{5})(\sqrt{5}-2)}{(5-\sqrt{5})(1+\sqrt{3})}.$$

IMAGINARY QUANTITIES.

Perform the operations indicated and simplify:

$$313. (a + b\sqrt{-1})(a - b\sqrt{-1}).$$

$$314. (a + b\sqrt{-1})^3.$$

$$315. \sqrt{-a^2 + 2ab - b^2}.$$

$$316. \sqrt{4a^2 - b^2 + 6bc - 9c^2 - 4(ab - 3ac)\sqrt{-1}}.$$

$$317. \frac{a + b\sqrt{-1}}{a - b\sqrt{-1}} + \frac{a - b\sqrt{-1}}{a + b\sqrt{-1}}.$$

$$318. \sqrt{4a^2 - 12ab\sqrt{-1} - 9b^2}.$$

$$319. (2\sqrt{-2})(3\sqrt{-3}).$$

$$320. (5 + \sqrt{-7})(5 - \sqrt{-7}).$$

$$321. (2\sqrt{8} - \sqrt{-10}) \div (-\sqrt{-2}).$$

$$322. \sqrt{2 + 4\sqrt{-42}}. \quad 323. \sqrt{7 + 6\sqrt{-2}}.$$

$$324. (4\sqrt{-3} + 7\sqrt{-2})(4\sqrt{-3} - 7\sqrt{-2}).$$

$$325. (\sqrt{-2} - 3\sqrt{-3})^2.$$

$$326. 3\sqrt{-4} - \sqrt{-25} + 4\sqrt{-9}.$$

$$327. 2\sqrt{-48} + 3\sqrt{-12} + 5\sqrt{-8} - 7\sqrt{-32}.$$

$$328. (2\sqrt{3} - \sqrt{-5})(4\sqrt{3} - 2\sqrt{-5}).$$

$$329. (a\sqrt{-1})^{4n}; (a\sqrt{-1})^{4n+2}; (a\sqrt{-1})^{4n+3}.$$

$$330. (5 - \sqrt{-2}) \div (1 + \sqrt{-2}).$$

$$331. [14 - \sqrt{15} - (7\sqrt{3} + 2\sqrt{5})\sqrt{-1}] \div (7 - \sqrt{5}\sqrt{-1}).$$

$$332. \sqrt{-2 - 2\sqrt{-15}}.$$

$$333. \{1 + \sqrt{-1}\}^2 - \{1 - \sqrt{-1}\}^2.$$

$$334. \{1 + \sqrt{-1}\}^2 + \{1 - \sqrt{-1}\}^2.$$

335. $\sqrt{+ \sqrt{-1}}.$

336. $\sqrt{- \sqrt{-1}}.$

337. $\sqrt{\left(\frac{a^2}{b^2} - d + \frac{2a\sqrt{d}}{b} \sqrt{-1}\right)}.$

338. $\sqrt{8 \sqrt{-1}}.$

EQUATIONS CONTAINING RADICAL EXPRESSIONS.

Solve the following equations :

339. $a + x = \sqrt{a^2 + x} \sqrt{b^2 + x^2}.$

340. $\frac{1}{2} \sqrt{x^2 + 3a^2} - \frac{1}{2} \sqrt{x^2 - 3a^2} = x \sqrt{a}.$

341. $\sqrt{a+x} + \sqrt{a-x} = b.$

342. $x + \sqrt{a^2 + x^2} = \frac{2a^2}{\sqrt{a^2 + x^2}}.$

343. $\sqrt{\left(\frac{x+1}{x-1}\right)} + \sqrt{\left(\frac{x-1}{x+1}\right)} = a.$

344. $\sqrt{a^2 + ax} = a - \sqrt{a^2 - ax}.$

345. $\sqrt{\left(\sqrt{\left(\sqrt{\left(\sqrt{x+123}\right)+4}\right)+5}\right)+6}+1=2.$

346. $\sqrt{2x + \sqrt{x^4 - x^2}} = x + 1.$

347. $\frac{\sqrt{x+28}}{4+\sqrt{x}} = \frac{38+\sqrt{x}}{\sqrt{x+6}}.$

348. $\sqrt{x} + \sqrt{x+2} = 4(2+x)^{-\frac{1}{2}}.$

349. $\sqrt{a+x} + \sqrt{a-x} = \sqrt{ax}.$

350. $b + x + \sqrt{b^2 + ax + x^2} = a.$

351. $\frac{\sqrt{x+2a}}{b+\sqrt{x}} = \frac{4a+\sqrt{x}}{\sqrt{x+3b}}.$

352. $1 + \sqrt{1+x} = \sqrt{1+x} + \sqrt{1-x}.$

$$353. \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}} = \sqrt{m}.$$

$$354. \frac{1+x^3}{(1+x)^2} + \frac{1-x^3}{(1-x)^2} = a.$$

$$355. \frac{\sqrt{x+16}}{\sqrt{x+4}} = \frac{\sqrt{x+32}}{\sqrt{x+12}} \quad 357. \frac{\sqrt{x+a}}{\sqrt{x+b}} = \frac{\sqrt{x+c}}{\sqrt{x+d}}.$$

$$356. \frac{\sqrt{x-8}}{\sqrt{x-6}} = \frac{\sqrt{x-4}}{\sqrt{x+2}} \quad 358. \sqrt{a^2+x^2} = \sqrt[4]{b^4+x^4}.$$

$$359. \sqrt{a^2+bx} + \sqrt{a^2-bx} = 2c.$$

$$360. x-4 = \frac{x^2}{(1+\sqrt{1+x})^2}.$$

$$361. 8\sqrt[3]{3x} + \frac{243+324\sqrt[3]{3x}}{16x-3} = 16x+3.$$

$$362. x-7 = \sqrt{49 + \sqrt{121x^2+x^4}}.$$

$$363. \sqrt[3]{a^3+x^3-2ax} = \frac{a+x}{\sqrt[3]{a+x}}.$$

$$364. \frac{1}{x} + \frac{1}{a} = \sqrt{\frac{1}{a^2}} + \sqrt{\frac{4}{b^2x^2}} + \frac{1}{x^4}.$$

$$365. \frac{(4x+1)^{\frac{1}{2}} + 2x^{\frac{1}{2}}}{(4x+1)^{\frac{1}{2}} - 2x^{\frac{1}{2}}} = 9.$$

$$366. \frac{x}{\sqrt{a^2+x^2}} = \frac{c-x}{\sqrt{b^2+(c-x)^2}}.$$

$$367. \sqrt{x-32} = 16 - \sqrt{x}.$$

$$368. \left(\frac{b}{a+x}\right)^{\frac{1}{2}} + \left(\frac{c}{a-x}\right)^{\frac{1}{2}} = \left(\frac{4bc}{a^2-x^2}\right)^{\frac{1}{2}}.$$

$$369. \sqrt{x+a} = c - \sqrt{x+b}.$$

$$370. x^{-1} + a^{-1} = \sqrt{a^{-2} + \sqrt{4a^{-2}x^{-2} + 9x^{-4}}}.$$

$$371. \frac{\sqrt{x+a} + \sqrt{x-a}}{\sqrt{x+a} - \sqrt{x-a}} = m.$$

$$372. \frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}} + \frac{x - \sqrt{x^2 - 1}}{x + \sqrt{x^2 - 1}} = 4x(x - 1).$$

$$373. x + \sqrt{x^2 - 2ax + b^2} = a + b.$$

$$374. \sqrt{x + 225} - \sqrt{x - 424} = 11.$$

$$375. \frac{\sqrt{4x+5} + \sqrt{x}}{\sqrt{4x+5} - \sqrt{x}} = 2.$$

$$376. \sqrt{4a^2 + x^2} = \sqrt[4]{4b^2 + x^4}.$$

$$377. \sqrt{x} + \sqrt{x-4} = \frac{8}{\sqrt{x-4}}.$$

$$378. \sqrt{x + \sqrt{x}} - \sqrt{x - \sqrt{x}} = \frac{3}{2} \sqrt{\frac{x}{x + \sqrt{x}}}.$$

$$379. \sqrt{\frac{x+4}{x-4}} + \sqrt{\frac{x-4}{x+4}} = \frac{10}{3}.$$

$$380. \frac{5\sqrt[3]{11x-17}}{4} + \frac{3}{8} = 4\frac{1}{8}.$$

$$381. \sqrt{\left(\frac{a^2}{x} + b\right)} - \sqrt{\left(\frac{a^2}{x} - b\right)} = c.$$

$$382. \sqrt[6]{x^2 + 11x + 5} = \sqrt[3]{x + 5}.$$

$$383. 3\sqrt[3]{3x-5} = 2\sqrt[3]{11x-17}.$$

$$384. \frac{x-4}{2+\sqrt{x}} = 5\sqrt{x} - 8 + \frac{3\sqrt{x}}{2}.$$

$$385. \frac{x-a}{\sqrt{a} + \sqrt{x}} = \frac{\sqrt{x} - \sqrt{a}}{3} + 2\sqrt{a}.$$

$$386. 1 - \sqrt{1-x} = n(1 + \sqrt{1-x}).$$

$$387. \sqrt[3]{a+x} + \sqrt[3]{a-x} = b.$$

$$388. \frac{1}{\sqrt{1-x}+1} + \frac{1}{\sqrt{1+x}-1} = \frac{1}{x}.$$

$$389. \sqrt[3]{(a+x)} + \sqrt[3]{(a-x)} = \sqrt[3]{b}.$$

$$390. \frac{a+x}{\sqrt{a}+\sqrt{a+x}} + \frac{a-x}{\sqrt{a}+\sqrt{a-x}} = \sqrt{a}.$$

$$391. \sqrt{1+x} + \sqrt{1+x+\sqrt{1-x}} = \sqrt{1-x}.$$

$$392. \frac{\sqrt{a} - \sqrt{a - \sqrt{a^2 - ax}}}{\sqrt{a} + \sqrt{a - \sqrt{a^2 - ax}}} = b.$$

V.

AFFECTED QUADRATICS.

Solve for values of x :

1. $x^2 - 7x + 3\frac{1}{4} = 0.$

3. $11\frac{3}{4}x - 3\frac{1}{2}x^2 = -41\frac{1}{4}.$

2. $x^2 - 5\frac{3}{4}x = 18.$

4. $9\frac{1}{2}x^2 - 90\frac{1}{2}x + 195 = 0.$

5. $6x^2 + 15x = 10x + 56.$

6. $4x^2 - 9x = 5x^2 - 255\frac{1}{4} - 8x.$

7. $\frac{x}{x+60} = \frac{7}{3x-5}.$

13. $\frac{5}{x+2} + \frac{3}{x} = \frac{14}{x+4}.$

8. $\frac{40}{x-5} + \frac{27}{x} = 13.$

14. $\frac{3x-2}{2x-5} - \frac{2x-5}{3x-2} = \frac{8}{3}.$

9. $\frac{8x}{x+2} - 6 = \frac{20}{3x}.$

15. $\frac{x-2}{x+2} + \frac{x+2}{x-2} = \frac{2(x+3)}{x-3}.$

10. $\frac{48}{x+3} = \frac{165}{x+10} - 5.$

16. $\frac{x^2-5x}{x+3} = x-3 + \frac{1}{x}.$

11. $\frac{x}{x+1} + \frac{x+1}{x} = \frac{13}{6}.$

17. $\frac{3}{5-x} + \frac{2}{4-x} = \frac{8}{x+2}.$

12. $\frac{1}{x-2} - \frac{2}{x+2} = \frac{3}{5}.$

18. $\frac{2x+3}{10-x} = \frac{2x}{25-3x} - 6\frac{1}{2}.$

19. $\frac{x-1}{x^2+x} - \frac{x+1}{x^2-x} = \frac{1}{4-2x}.$

20. $32a^{2m}c^{n-1} + 4a^{m+3}c^{n-1}(ac^3-2)x = a^7c^{n+2}x^2.$

$$21. x + \frac{24}{x-1} = 3x - 4. \quad 22. \frac{4x}{5-x} - \frac{4(5-x)}{x} = 15.$$

$$23. \frac{3x-7}{x} = 3\frac{1}{2} - \frac{4(x-2\frac{1}{2})}{x+5}.$$

$$24. \frac{7+x}{7-x} + \frac{7-x}{7+x} = 2\frac{9}{10}.$$

$$25. \frac{3x+2}{3x-2} + \frac{3x-2}{3x+2} = \frac{15x+11}{3x+2}.$$

$$26. \frac{65x}{2} - \frac{10x^2}{11} = \frac{13}{2} - \frac{2x}{11}. \quad 28. \frac{4x-3}{3x-7} - \frac{2x-3}{x-1} = 3.$$

$$27. \frac{x}{x+8} = \frac{x+3}{2x+1}. \quad 29. \frac{3x-5}{3x+5} + \frac{135}{176} = \frac{3x+5}{3x-5}.$$

$$30. \frac{x+8}{x+12} + \frac{5}{x+4} = \frac{3x+14}{3x+8}.$$

$$31. 2\left(\frac{x-1}{x+1}\right) + \frac{x+1}{x-1} = \frac{17}{6}.$$

$$32. -x^2 + x = \frac{6}{25}.$$

$$33. 3x^2 - 30x = 9(x-12).$$

$$34. \frac{x}{2} + \frac{2}{x} = \frac{x}{4} + \frac{3}{2}.$$

$$35. adx - acx^2 = bcx - bd.$$

$$36. \sqrt[4]{x+13} + 5\sqrt{x+13} = 22.$$

$$37. \frac{3}{4}(x^2 - 3) = \frac{1}{8}(x - 3).$$

$$38. x^2 + (x+1)^2 = \frac{13}{8}x(x+1).$$

$$39. a^2x^2 - 2a^3x + a^4 - 1 = 0.$$

$$40. 4(x-1) - \frac{x-1}{2x} = 3\frac{3}{4}.$$

$$41. 5\frac{1}{2} - \frac{2}{x} = \frac{7}{x+1}.$$

$$42. a^2x^2 + abx = acx + bc.$$

$$43. \frac{1}{2}x^2 + 5 = \frac{2}{3}x + 5\frac{5}{8}.$$

$$44. 7x^2 - 2 = -(2 - \sqrt{3})x + 4x^2\sqrt{3}.$$

$$45. \frac{5-x}{3+x} - \frac{x}{3} = \frac{1}{8}x - \frac{7+4x}{19}.$$

$$46. \frac{x}{m} + \frac{m}{x} = \frac{5}{m}.$$

$$47. mx^2 + mn = 2mx\sqrt{n} + nx^2.$$

$$48. (1+x+x^2)^2 = \frac{(a+1)(1+x^2+x^4)}{a-1}.$$

$$49. \frac{x^4 + 3x^3 + 6}{x^2 + x - 4} = x^2 + 2x + 15.$$

$$50. \frac{31}{6x} = \frac{16}{117-2x} + 1.$$

$$51. abx^2 + \frac{3a^2x}{c} = \frac{6a^2 + ab - 2b^2}{c^2} - \frac{b^2x}{c}.$$

$$52. \frac{3x}{x+2} - \frac{x-1}{6} = x - 2\frac{1}{2}.$$

$$53. 4a^2x = (a^2 - b^2 + x)^2.$$

$$54. \frac{x + \sqrt{x}}{x - \sqrt{x}} = \frac{x^2 - x}{4}.$$

$$55. \frac{x}{3} + \frac{3}{x} = \frac{x}{4} + \frac{4}{x} - \frac{11}{12}.$$

$$56. \frac{x-2}{x+2} - \frac{x-3}{x+3} = \frac{x+4}{x-4} - \frac{x+2}{x-2}.$$

$$57. (7x+3)(3+7x) = 10\{2(x-1)(3+x) - (3+2x)(x-3)\}.$$

$$(a-m+x)^{-1} = a^{-1} - m^{-1} + x^{-1}.$$

$$59. abx^2 - 2x(a+b)\sqrt{ab} = (a-b)^2.$$

$$60. \frac{25x+180}{10x-81} = \frac{40x}{5x-8} - \frac{3}{5}.$$

$$61. \frac{x^2+a^2}{a-x} - \frac{x}{2} + \frac{3}{2}(x - \frac{1}{2}a) + \frac{1}{4}a = 2a - x.$$

SOLUTION OF EQUATIONS SOLVED LIKE QUADRATICS.

$$62. x^4 + 4x^3 + 4x + 1 = \frac{57x^2}{4}.$$

$$63. cx^2 - 2cx\sqrt{d} = dx^2 - cd.$$

$$64. \frac{x^2+1}{x} + \frac{7}{2} \cdot \frac{x+1}{\sqrt{x}} = 13.$$

$$65. \frac{2x+\sqrt{x}}{2x-\sqrt{x}} + 3\left(\frac{2x-\sqrt{x}}{2x+\sqrt{x}}\right) = 4.$$

$$66. (x-4)^2 + 2(x-4) = \frac{2}{x} - 1.$$

$$67. x-3 = \frac{3+4\sqrt{x}}{x}.$$

$$68. x^2 - x + 5\sqrt{2x^2 - 5x + 6} = \frac{3x+33}{2}.$$

$$69. \{(x-2)^2 - x\}^2 - (x-2)^2 = 90 - x.$$

$$70. \frac{\sqrt{(x+2a)} - \sqrt{(x-2a)}}{\sqrt{(x-2a)} + \sqrt{(x+2a)}} = \frac{x}{2a}.$$

$$71. x^4 - 2x^3 - 2x^2 + 3x = 108.$$

$$72. (x+2)^2 + (x+2) = 20.$$

$$41. 5\frac{1}{2} - \frac{2}{x} = \frac{7}{x+1}.$$

$$42. a^2x^2 + abx = acx + bc.$$

$$43. \frac{1}{2}x^2 + 5 = \frac{2}{3}x + 5\frac{5}{8}.$$

$$44. 7x^2 - 2 = -(2 - \sqrt{3})x + 4x^2\sqrt{3}.$$

$$45. \frac{5-x}{3+x} - \frac{x}{3} = \frac{1}{8}x - \frac{7+4x}{19}.$$

$$46. \frac{x}{m} + \frac{m}{x} = \frac{5}{m}.$$

$$47. mx^2 + mn = 2mx\sqrt{n} + nx^2.$$

$$48. (1+x+x^2)^2 = \frac{(a+1)(1+x^2+x^4)}{a-1}.$$

$$49. \frac{x^4 + 3x^3 + 6}{x^2 + x - 4} = x^2 + 2x + 15.$$

$$50. \frac{31}{6x} = \frac{16}{117-2x} + 1.$$

$$51. abx^2 + \frac{3a^2x}{c} = \frac{6a^2 + ab - 2b^2}{c^2} - \frac{b^2x}{c}.$$

$$52. \frac{3x}{x+2} - \frac{x-1}{6} = x - 2\frac{1}{2}.$$

$$53. 4a^2x = (a^2 - b^2 + x)^2.$$

$$54. \frac{x + \sqrt{x}}{x - \sqrt{x}} = \frac{x^2 - x}{4}.$$

$$55. \frac{x}{3} + \frac{3}{x} = \frac{x}{4} + \frac{4}{x} - \frac{11}{12}.$$

$$56. \frac{x-2}{x+2} - \frac{x-3}{x+3} = \frac{x+4}{x-4} - \frac{x+2}{x-2}.$$

$$57. (7x+3)(3+7x) = 10\{2(x-1)(3+x) - (3+2x)(x-3)\}.$$

$$58. (a-m+x)^{-1} = a^{-1} - m^{-1} + x^{-1}.$$

$$59. abx^2 - 2x(a+b)\sqrt{ab} = (a-b)^2.$$

$$60. \frac{25x+180}{10x-81} = \frac{40x}{5x-8} - \frac{3}{5}.$$

$$61. \frac{x^2+a^2}{a-x} - \frac{x}{2} + \frac{3}{2}(x - \frac{1}{2}a) + \frac{1}{4}a = 2a - x.$$

SOLUTION OF EQUATIONS SOLVED LIKE QUADRATICS.

$$62. x^4 + 4x^3 + 4x + 1 = \frac{57x^2}{4}.$$

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$$68. x^2 - x + 5\sqrt{2x^2 - 5x + 6} = \frac{3x+33}{2}.$$

$$69. \{(x-2)^2 - x\}^2 - (x-2)^2 = 90 - x.$$

$$70. \frac{\sqrt{(x+2a)} - \sqrt{(x-2a)}}{\sqrt{(x-2a)} + \sqrt{(x+2a)}} = \frac{x}{2a}.$$

$$71. x^4 - 2x^3 - 2x^2 + 3x = 108.$$

$$72. (x+2)^2 + (x+2) = 20.$$

$$73. \frac{x}{x+4} + \frac{4}{\sqrt{x+4}} = \frac{21}{x}. \quad 74. x^4 - 8x^2 - 24x = 32.$$

$$75. \frac{x}{\sqrt{x} + \sqrt{a-x}} + \frac{x}{\sqrt{x} - \sqrt{a-x}} = \frac{b}{\sqrt{x}}.$$

$$76. \frac{x + \sqrt{x^2 - 9}}{x - \sqrt{x^2 - 9}} = (x - 2)^2.$$

$$77. x^n - 2ax^{\frac{n}{2}} = b.$$

$$78. \frac{123 + 41\sqrt{x}}{5\sqrt{x-x}} = \frac{4(5\sqrt{x+x})}{3-\sqrt{x}} - \frac{2x^2}{(5\sqrt{x-x})(3-\sqrt{x})}.$$

$$79. \frac{x+4}{x-3} - \frac{2x-3}{x+4} = 7\frac{3}{8}. \quad 80. \sqrt{x^5} + \sqrt{x^3} = 6\sqrt{x}.$$

$$81. \sqrt[4]{x+21} = 12 - \sqrt{x+21}.$$

$$82. \sqrt{x^3} - 2\sqrt{x} - x = 0.$$

$$83. \frac{x^5 + x^4 + 2}{x^5 - x^4} = \frac{x^3 + x^2 - 2}{x^3 - x^2}.$$

$$84. \frac{\sqrt{a+x}}{\sqrt{a} + \sqrt{a+x}} = \frac{\sqrt{a-x}}{\sqrt{a} - \sqrt{a-x}}.$$

$$85. \left(\frac{x}{x-1}\right)^2 + \left(\frac{x}{x+1}\right)^2 = n(n-1).$$

$$86. x + 4 + \left(\frac{x+4}{x-4}\right)^{\frac{1}{2}} = \frac{12}{x-4}.$$

$$87. \frac{\sqrt{x^2-16}}{\sqrt{x-3}} + \sqrt{x+3} = \frac{7}{\sqrt{x-3}}.$$

$$88. x + 5 = 6 + \sqrt{x+5}.$$

$$89. x^2 + \frac{1}{x^2} - a^2 - \frac{1}{a^2} = 0.$$

$$90. \frac{\sqrt{x^2+1} + \sqrt{x^2-1}}{\sqrt{x^2+1} - \sqrt{x^2-1}} + \frac{\sqrt{x^2+1} - \sqrt{x^2-1}}{\sqrt{x^2+1} + \sqrt{x^2-1}} = 4\sqrt{x^2-1}.$$

$$91. \frac{x^3-4x}{x-2} + \frac{x^2-1}{x+1} = 39.$$

$$92. cx + \frac{ac}{a+b} = (a+b)x^2.$$

$$93. x(x+1) + 3\sqrt{2x^2+6x+5} = 25-2x.$$

$$94. x^2-7+\sqrt{3x^2-27} = 110-2x^2.$$

$$95. x^2 + \frac{2}{3}\sqrt{3x^2+3x} = 16-x.$$

$$96. x^2-3x+7\sqrt{11x-2x^2+2} = \frac{5x}{2} + 21.$$

$$97. x^2+x+2\sqrt{x^2+x+4} = 20.$$

$$98. (4a^2-9cd^2)x^2 + (4a^2c^2+4abbd^2)x + (ac^2+bd^2)^2 = 0.$$

$$99. \frac{x}{4} = \frac{\sqrt{x-12}}{x-18}. \quad 102. x^3-8x^2+11x = -20.$$

$$100. x^3-6x^2+11x = 6. \quad 103. \frac{x+a}{x+b} = \left(\frac{2x+a+c}{2x+b+c}\right)^2.$$

$$101. x^3-4x^2+x+6 = 6. \quad 104. 3x^3-14x^2+21x = 10.$$

$$105. x+a+3\sqrt{abx} = b.$$

$$106. 9x-4x^2+(4x^2-9x+11)^{\frac{1}{2}} = 5.$$

$$107. 5x-7x^2+8\sqrt{7x^2-5x+1} = 8.$$

$$108. \frac{x^2+b^2}{x^2-b^2} + \frac{2x+3b-a}{x+b} = \frac{2(x^2+bx-b^2)}{x^2-b^2}.$$

$$109. \frac{x^2+x+\frac{1}{2}}{a^2+1} + \frac{x^2+x}{a^2-1} = 0.$$

$$110. (x^2+1)(x+2) = 2. \quad 111. ax^{2n}+bx^n = c.$$

$$112. \sqrt{x} - \frac{8}{x} = \frac{7}{\sqrt{x} - 2}.$$

$$113. \frac{54 - 9\sqrt{x}}{x + 2\sqrt{x}} = \frac{7x^2 - 3x + 4}{(6 + \sqrt{x})(x + 2\sqrt{x})} + \frac{23x - 46\sqrt{x}}{6 + \sqrt{x}}.$$

$$114. x^3 - 3x^2 + 3x = 9.$$

$$115. \sqrt{(x-1)(x-2)} + \sqrt{(x-3)(x-4)} = \sqrt{2}.$$

$$116. \sqrt{x^2 + ax + b} + \sqrt{x^2 - ax + b} = c.$$

$$117. \frac{1}{5} \left\{ \frac{(x+1)(x-3)}{(x+2)(x-4)} \right\} + \frac{1}{9} \cdot \frac{(x+3)(x-5)}{(x+4)(x-6)} \\ - \frac{2}{13} \cdot \frac{(x+5)(x-7)}{(x+6)(x-8)} = \frac{92}{585}.$$

SIMULTANEOUS EQUATIONS CONTAINING QUADRATICS.

Solve the following equations for values of x and y :

$$118. \begin{cases} x + y + \sqrt{x+y} = 12, \\ xy = 20. \end{cases}$$

$$119. \begin{cases} 2x^2 - 3xy + 4y^2 = 24, \\ 3x^2 - 5y^2 = 28. \end{cases}$$

$$120. \begin{cases} x^5 + y^5 = 33, \\ x + y = 3. \end{cases}$$

$$124. \begin{cases} x^3 - y^3 = 26, \\ x - y = 2. \end{cases}$$

$$121. \begin{cases} x^3 + y^3 = 91, \\ x + y = 7. \end{cases}$$

$$125. \begin{cases} x + y = 4, \\ x^3 + y^3 = (x+y)^2. \end{cases}$$

$$122. \begin{cases} 3x^2 - 2y^2 = 115, \\ 2x - 3y = 2. \end{cases}$$

$$126. \begin{cases} \sqrt[3]{x} + \sqrt[3]{y} = 3, \\ \sqrt[3]{xy} = 2. \end{cases}$$

$$123. \begin{cases} 4x^2 + 3y^2 = 511, \\ 3x + 2y = 27. \end{cases}$$

$$127. \begin{cases} x + 4y = 14, \\ y^2 + 4x = 2y + 11. \end{cases}$$

$$128. \begin{cases} x^2 + xy = 84, \\ x^2 - y^2 = 24. \end{cases}$$

$$132. \begin{cases} 2x + 3y = 118, \\ 5x^2 - 7y^2 = 4333. \end{cases}$$

$$129. \begin{cases} x^3 + y^3 = 152, \\ x^2 - xy + y^2 = 19. \end{cases}$$

$$133. \begin{cases} \frac{18x}{y} = \frac{8y}{x}, \\ 3xy + 2x + y = 485. \end{cases}$$

$$130. \begin{cases} x^3 - y^3 = 7xy, \\ x - y = 2. \end{cases}$$

$$134. \begin{cases} x^2y + xy^2 = 30, \\ \frac{1}{x} + \frac{1}{y} = \frac{5}{6}. \end{cases}$$

$$131. \begin{cases} \frac{x^2}{y} - \frac{y^2}{x} = 8\frac{2}{3}, \\ x - y = 2. \end{cases}$$

$$135. \begin{cases} x^2 + y^2 - x - y = 78, \\ xy + x + y = 39. \end{cases}$$

$$136. \begin{cases} 2x + y = 26 - 7\sqrt{2x + y + 4}, \\ \frac{2x + \sqrt{y}}{2x - \sqrt{y}} = \frac{16}{15} + \frac{2x - \sqrt{y}}{2x + \sqrt{y}}. \end{cases}$$

$$137. \begin{cases} x + y + x^2 + y^2 = a, \\ x - y + x^2 - y^2 = b. \end{cases}$$

$$140. \begin{cases} x^2 - xy + y^2 = 7, \\ x^4 + x^2y^2 + y^4 = 133. \end{cases}$$

$$138. \begin{cases} 2y^2 = xy + 2, \\ 4x^2 = xy + 30. \end{cases}$$

$$141. \begin{cases} x^2 + xy + y^2 = 49, \\ x^4 + x^2y^2 + y^4 = 931. \end{cases}$$

$$139. \begin{cases} x^2 - x^2y^2 + y^2 = 19, \\ x - xy + y = 4. \end{cases}$$

$$142. \begin{cases} x^4 - x^2 + y^4 - y^2 = 84, \\ x^2 + x^2y^2 + y^2 = 49. \end{cases}$$

$$143. \begin{cases} x^{\frac{1}{2}} + y^{\frac{1}{2}} = 6, \\ x^{\frac{2}{3}} + y^{\frac{2}{3}} = 126. \end{cases}$$

$$144. \begin{cases} \frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{x}{y} + \frac{y}{x} = \frac{27}{4}, \\ x^2 + y^2 = 20. \end{cases}$$

$$145. \begin{cases} 3x^2 + 4y^2 = 7xy, \\ x^{\frac{3}{2}} - \frac{2y^2}{9} = yx^{\frac{1}{2}}. \end{cases}$$

$$146. \begin{cases} \frac{x}{y} - 8\sqrt{x^2 - 9xy^2} = 9y - 16xy, \\ 5x = 4 + 25y^2. \end{cases}$$

$$147. \begin{cases} 16x - y^{\frac{1}{2}} = 6x^{\frac{1}{2}}y^{\frac{1}{2}}, \\ \frac{x^{\frac{1}{2}}}{y} - \frac{12}{x^2} = \frac{x}{\sqrt{y}}. \end{cases}$$

$$148. \begin{cases} (x-y)(x^2-y^2) = 160, \\ (x+y)(x^2+y^2) = 580. \end{cases}$$

$$149. \begin{cases} x^4 + y^4 = x, \\ x^3 + y^3 = 1. \end{cases}$$

$$150. \begin{cases} \sqrt{5\sqrt{x} + 5\sqrt{y}} + \sqrt{y} = 10 - \sqrt{x}, \\ \sqrt{x^5} + \sqrt{y^5} = 275. \end{cases}$$

$$151. \begin{cases} y^2 - x^2 - y - x = 12, \\ (y-x)^2 (y+x) = 48. \end{cases}$$

$$152. \begin{cases} x^2 + y^2 = 34, \\ x^2 - xy = 10. \end{cases}$$

$$158. \begin{cases} x + y = 10, \\ \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{5}{2}. \end{cases}$$

$$153. \begin{cases} x^2 + 3xy + 4y^2 = 14, \\ 3x^2 + 4xy + 5y^2 = 25. \end{cases}$$

$$154. \begin{cases} \frac{y}{x} + \frac{3x}{x+y} = \frac{x^2-y^2}{y}, \\ \frac{x}{y} - \frac{x+y}{x} = \frac{y}{x}. \end{cases} \quad 159. \begin{cases} x + 2y + \frac{3x}{y} = 16, \\ 3x + y + \frac{3x}{y} = 23. \end{cases}$$

$$155. \begin{cases} \frac{4x}{5y} = \frac{14}{15}, \\ x^2 + y^2 - xy - 7y = 1 \end{cases}$$

$$160. \begin{cases} x^{\frac{1}{2}} + y^{\frac{1}{2}} = 5, \\ x^{\frac{2}{3}} + y^{\frac{2}{3}} = 13. \end{cases}$$

$$156. \begin{cases} \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{10}{3}, \\ x^2 + y^2 = 45. \end{cases}$$

$$161. \begin{cases} x - y = 208, \\ \sqrt[3]{x} - \sqrt[3]{y} = 4. \end{cases}$$

$$162. \begin{cases} x^2 + xy = 77, \\ xy - y^2 = 12. \end{cases}$$

$$157. \begin{cases} \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{5}{2}, \\ x^2 + y^2 = 20. \end{cases}$$

$$163. \begin{cases} \frac{x^2}{y} + \frac{y^2}{x} = 18, \\ x + y = 12. \end{cases}$$

$$164. \begin{cases} x^2y^4 - 7xy^2 - 945 = 765, \\ xy - y = 12. \end{cases}$$

$$165. \begin{cases} x - 2\sqrt{xy} + y - \sqrt{x} + \sqrt{y} = 0, \\ \sqrt{x} + \sqrt{y} = 5. \end{cases}$$

$$166. \begin{cases} \frac{x^2}{y^2} + \frac{4x}{y} = \frac{85}{9}, \\ x - y = 2. \end{cases}$$

$$167. \begin{cases} \sqrt{\frac{3x}{x+y}} + \sqrt{\frac{x+y}{3x}} = 2, \\ xy - (x+y) = 54. \end{cases}$$

$$168. \begin{cases} x^4 - 2x^2y + y^2 = 49, \\ x^4 - 2x^2y^2 + y^4 - x^2 + y^2 = 20. \end{cases}$$

PROBLEMS PRODUCING QUADRATIC EQUATIONS.

169. A and B engaged to cradle a field of grain for \$36, and as A alone could cradle it in 18 days, they promised to complete it in 10 days. They were obliged, however, to call in C, an inferior workman, to assist them for the last four days, in consequence of which B received \$1.50 less than he otherwise would have. In what time could B or C separately reap the field?

170. A gardener undertook to plant a certain number of trees at equal distances apart, and in the form of a square. In the first attempt when he had finished his square he had 11 trees to spare. He then added one of these to each row as far as they would go, and found that he wanted 24 trees more to complete this square. How many trees had he?

171. A farmer paid \$240 for a certain number of sheep, out of which he reserved 15, and sold the remainder for \$216, gaining 40 cts. a head on those he sold. How many sheep did he buy, and what was the price of each?

172. A set out from W. to C.; B at same time from C. to W. A arrived in C. 9 hours, and B in W. 16 hours, after they had met? In what time did each perform the journey?

173. A courier proceeds from P. to Q. in 14 hours; a second courier starts at same time from a place 10 miles behind P. and arrives at Q. at the same time as the first courier. The second courier finds that he takes half an hour less than the first to accomplish 20 miles. Find the distance of Q. from P.

174. A person, who can walk forwards four times as fast as he can backwards, undertakes to walk a certain distance, one-fourth of it backwards, in a stated time. He finds that if his speed per hour backwards were one-fifth of a mile less he would need to walk forwards two miles an hour faster to gain his object. What is his speed?

175. A boat's crew row down a river $3\frac{1}{2}$ miles and back again in 1 hour 40 minutes. Supposing the river to have a current of 2 miles an hour, find the rate at which the crew would row in still water?

176. A vintner draws a certain quantity of wine out of a full vessel that contains 256 gallons, and then filling the vessel with water, draws out the same quantity as before, and so on for four draughts, after which there are only 81 gallons of pure wine left. How much wine does he draw each time?

177. A tradesman buys cloth and has to pay for insurance and carriage 4 per ct.; he sells it for 390 dollars, and gains as much per cent. as the number of dollars in the 12th part of the cost. What does he give for the cloth?

178. During the time that the shadow of a sun-dial, which shows true time, moves from one o'clock to five, a clock, which is too fast by a certain number of hours and minutes, strikes in total a number of strokes which is equal to the sum of the number of hours and number of minutes by which it is too fast, and it is observed that the number of minutes is less by

41 than the square of the number which the clock strikes at the last time of striking. The clock does not strike 12 during the time. How much is it too fast?

179. A person rows 20 miles down the river and back again in 10 hours, and he finds that he can row 2 miles against the stream in the same time that he can row 3 miles with it. Required the rate of the stream, and the times of his going and returning.

180. A wine merchant sold 7 doz. sherry and 12 doz. claret for £50. Of sherry he sold 3 doz. more for £10 than he did of claret for £6. What was the price of each per doz.?

181. A person having travelled 56 miles on a railroad and the rest of his journey by a coach, observed that in the train he had performed $\frac{1}{4}$ of his whole journey in the time the coach took to go 5 miles, and that at the instant he arrived at home the train must have reached a point 35 miles farther than he was from the station at which it left him. Compare the rates of the coach and the train.

182. A cloth being wet shrinks $\frac{1}{8}$ in its length and $\frac{1}{16}$ in its width. If the surface of a piece of similar cloth is diminished by $5\frac{3}{4}$ square yards, and the length of the four sides by $4\frac{1}{4}$ yards, what is the original length and width of the cloth?

183. A and B set out at the same time, A from C. to go to D., and B from D. to go to C.; they meet on the road, when it appears that A has travelled 30 miles more than B, and that at the rate he is travelling he will reach D. in 4 days, and that B will arrive in C. in 9 days. Find the distance of C. from D.

184. Divide a number m into two parts in such a manner that the product of the two parts may be equal to n .

185. There are 2 rectangular vats, the greater of which contains 20 cubic feet more than the other. Their capacities are in the ratio of 4 to 5, and their bases are squares, a side

of each of which is equal to the depth of the other vat. What are the depths?

186. The product of 4 consecutive numbers is 840. Find them.

187. A and B travelled on the same road and at the same rate to London. At the 50th mile-stone from London A overtook a flock of geese, which were travelling at the rate of 3 miles in 2 hours, and 2 hours afterward he met a wagon which was travelling at the rate of 9 miles in 4 hours. B overtook the flock of geese at the 45th mile-stone from London, and met the wagon 40 minutes before he came to the 31st mile-stone. Where was B when A reached London?

188. A ladder whose foot rests in a given position just reaches a window on one side of a street, and when turned about its foot just reaches a window on the other side. If the two positions of the ladder be at right angles to each other, and the heights of the windows be 36 and 27 feet respectively, find the width of the street and the length of the ladder.

THEORY OF EQUATIONS.

Find all possible real roots of the following equations :

189. $x^2 - 42x + 117 = 0$.

190. $5x^2 - 15ax - 2bx + 6ab = 0$.

191. $(x^2 + 5ax + 6a^2)(x^2 - 7ax + 12a^2) = 0$.

192. $(x^2 - 4)(x^2 - 2ax + a^2) = 0$.

193. $x(x^2 - 5x) = 0$.

194. $(acx - 2a + b)(bcx + 3a - b) = 0$.

195. $x^3 - 9x^2 + 26x - 24 = 0$.

$$196. x^3 - 49x - 120 = 0.$$

$$197. x^3 - 8x^2 + 5x + 14 = 0.$$

$$198. x^3 - 18x^2 + 87x - 110 = 0.$$

$$199. x^3 - 13x^2 + 49x - 45 = 0.$$

$$200. x^4 + 29x^3 + 287x^2 + 1147x + 1560 = 0.$$

$$201. x^4 - 10x^3 + 35x^2 - 50x + 24 = 0.$$

$$202. x^4 - 45x^2 - 40x + 84 = 0.$$

$$203. x^3 + \frac{17}{4}x^2 - \frac{79}{8}x + \frac{15}{4} = 0.$$

$$204. x^3 - 6x^2 + 19x - 44 = 0.$$

$$205. x^5 - 3x^4 - 8x^3 + 24x^2 - 9x + 27 = 0.$$

$$206. x^4 - \frac{41}{8}x^3 + \frac{287}{32}x^2 - \frac{393}{64}x + \frac{45}{32} = 0.$$

$$207. x^3 - \frac{13}{2}x^2 + \frac{8}{3}x - \frac{1}{24} = 0.$$

$$208. x^2 + 5x + 28 - 5\sqrt{(x^2 + 5x + 28)} + \frac{25}{4} = \frac{121}{4}.$$

$$209. x^4 - 20x^2 + 64 = 0.$$

$$210. x^4 - 2x^3 + 3x^2 - 2x - 3 = 0.$$

$$211. x^2 - 7x + \sqrt{(x^2 - 7x + 18)} = 24.$$

$$212. x^4 + \frac{3x^3}{2} - 24x - 256 = 0.$$

$$213. x^4 + \frac{13x^3}{3} - 39x - 81 = 0.$$

$$214. x^4 - \frac{2x^3}{3} - \frac{80x^2}{9} + 6x - 1 = 0.$$

$$215. x^{2m} + 3x^m = 4.$$

$$216. x^{4n} - \frac{5}{8}x^{2n} = \frac{25}{12}.$$

$$218. x^{-2} + 3x^{-1} = \frac{4}{3}.$$

$$217. x - \frac{9}{2}x^{\frac{1}{2}} = \frac{5}{2}.$$

$$219. x^{-2n} - x^{-n} = 20.$$

Resolve the following quadratic expressions into simple factors:

220. $3x^2 - 10x - 25.$

226. $4x^3 + 6x^2 + x - 1.$

221. $2x^2 + x - 6.$

227. $6x^3 + 11x^2 - 9x - 14.$

222. $2x^3 + x^2 - 11x - 10.$

228. $x^3 + y^3 + z^3 - 3xyz.$

223. $x^3 - 11x^2 + 36x - 36.$

229. $3x^3 - x^2 - 23x + 21.$

224. $x^3 - 7x^2 + 14x - 8.$

230. $2x^3 - 5x^2 - 17x + 20.$

225. $x^3 - 5x^2 - 46x - 40.$

231. $15x^3 + 41x^2 + 5x - 21.$

232. $x^4 + \frac{3}{2}x^3 - \frac{3}{2}x^2 - \frac{3}{2}x + 1.$

233. $x^3 + \frac{4}{3}x^2 + x - \frac{8}{3}.$

234. $x^3 - (\frac{3}{2} - \sqrt{3})x^2 - (\frac{3}{2}\sqrt{3} - 2)x + 2\sqrt{3}.$

235. $x^4 + \frac{3}{2}x^3 - 18x^2 - \frac{5}{2}x + 42.$

Form equations whose roots are as follows:

236. 4 and 5.

241. $\frac{1}{2}$ and $\frac{2}{3}.$

237. $\frac{1}{2}$ and $-3.$

242. 7 and $-\frac{5}{9}.$

238. -3 and $4.$

243. $\sqrt{3}$ and $-\sqrt{3}.$

239. 4 and $-5.$

244. 1, -1 , 3 , -2 , and $2 \pm \sqrt{7}.$

240. -2 and $-7.$

245. 2, -2 , 3 , and $0.$

246. $1 \pm \sqrt{5}.$

247. Given 3, one root of the equation $x^4 - 4x^3 + 8x^2 - 8x = 21$, to find the other roots.

248. Given 3 and -4 , two roots of the equation $x^5 - 2x^4 - 25x^3 + 26x^2 + 120x = 0$, to find the other roots.

249. For what value of c will the equation $2x^2 + 4x + c = 0$ have equal roots?

250. If m and n are the roots of the equation $ax^2 + bx + c = 0$, form the equation whose roots are the reciprocals of these.

251. If m and n are the roots of the equation $x^2 + px + q = 0$, find the value of $m^2 + n^2$, of $(m - n)^2$, of $m^2 - n^2$, of $\frac{1}{m} + \frac{1}{n}$, and of $m^3 - n^3$.

BINOMIAL THEOREM.

Formula: to find the $(r + 1)$ th term of $(ax + by)^n$.

$$\frac{\underline{n} \dots (n - r + 1) \cdot a^{n-r} b^r}{\underline{r}} x^{n-r} y^r.$$

\underline{n} means "factorial n ;" e. g.: $\underline{6} = 6 \times 5 \times 4 \times 3 \times 2 \times 1$.

$\underline{n} \dots (n - r + 1)$ means stop with the number $(n - r + 1)$;
e. g.: $\underline{10} \dots 8 = 10 \times 9 \times 8$.

For example: find 5th term of $(3x + 4y)^7$.

$$n = 7, r = 4, a = 3, b = 4.$$

$$\frac{7 \times 6 \times 5 \times 4 \times 3^3 \times 4^4}{4 \times 3 \times 2 \times 1} x^3 y^4 = 241920 x^3 y^4 = 5\text{th term.}$$

Expand the following expressions:

252. $(5 + 4x)^4$.

257. $(3y^2 - \frac{1}{2}z^3)^4$.

253. $(3 - 2x^2)^6$.

258. $(2x - 3y)^5$.

254. $\left(\frac{x}{2} + 2y\right)^6$.

259. $\left(x - \frac{1}{x}\right)^7$.

255. $(3ac - 2bd)^5$.

256. $\left(\frac{2x^3}{y^2} + 3z^2\right)^3$.

260. $\left(2y - \frac{x^2}{z^3}\right)^4$.

Expand to five terms the following:

261. $(1 - \frac{1}{2}x)^{-5}$.

266. $(a - x^2)^{-3}$.

262. $\frac{1}{(1 - 2x)^5}$.

267. $(a^2 + x^3)^{-1}$.

268. $(a^{\frac{1}{2}} - x^{-\frac{1}{2}})^{-\frac{1}{2}}$.

263. $\frac{1}{(1 - x)^4}$.

269. $(a^2 m - x^{\frac{1}{2}})^{-\frac{2}{3}}$.

264. $(1 + \frac{2}{3}x)^{\frac{1}{2}}$.

270. $\frac{1}{\sqrt{a - bx}}$.

265. $\frac{1}{(1 - x)^{\frac{4}{3}}}$.

271. $\frac{1}{(a + x^{-3})^{-\frac{2}{3}}}$.

Expand to four terms the following:

272. $(a^5 + a^3 x^2)^{\frac{2}{3}}$.

279. $(a^{\frac{1}{2}} + x^{\frac{1}{2}})^{\frac{1}{2}}$.

273. $(a^2 + ax)^{\frac{1}{16}}$.

280. $(1 - a^2)^{\frac{2}{3}}$.

274. $(1 - \frac{5}{8}x)^{-\frac{3}{2}}$.

281. $(1 - 3x)^{\frac{3}{4}}$.

275. $\sqrt[3]{(a^3 - x)}$.

282. $\left(x^2 - \frac{2y}{3}\right)^{\frac{3}{2}}$.

276. $\sqrt[3]{(1 - x^3)}$.

283. $\frac{a^2}{(a + x)^2}$.

277. $\sqrt[3]{6}$ or $\sqrt[3]{(8 - 2)}$.

284. $\left(\frac{2x}{3} - \frac{3y}{2}\right)^{\frac{2}{3}}$.

278. $\left(a + \frac{4x}{3}\right)^{\frac{3}{4}}$.

Find terms in the expanded series as follows:

285. The fourth term of $(a - b)^{100}$.

286. The ninth term of $(2ab - cd)^{14}$.

287. The middle term of $(a - b)^{16}$.

288. The middle term of $(a^{\frac{1}{2}} + b^{\frac{1}{2}})^8$.

289. The two middle terms of $(a - b)^{19}$.

290. The two middle terms of $(a + x)^{13}$.

291. The sixth term of $(x^3 + 3xy)^9$.

292. The third term of $(1 - 2y)^7$.

293. The third term of $(1 - \frac{5}{6}x)^{-\frac{3}{2}}$.

294. The third term of $(1 - 2x)^{-1}$.

295. The third term of $(3 - 2x^2)^5$.

296. The fifth term of $(3y - 4z)^8$.

297. The fourth term of $\left(1 - \frac{3y}{2z}\right)^6$.

298. The sixth term of $(1 + x)^{-\frac{3}{2}}$.

299. The sixth term of $(a + \frac{1}{2}x)^{\frac{5}{2}}$.

300. The fourth term of $\frac{a^2}{(a^2 - x)^{\frac{1}{2}}}$.

301. The fourth term of $\sqrt[3]{9}$ or $(8 + 1)^{\frac{1}{3}}$.

302. The fourth term of $\frac{1}{(a - x)^{\frac{1}{2}}}$.

303. The eighth term of $(\frac{4}{3}x + \frac{2}{3}y)^7$.

304. The eighth term of $(3x^2 - 2\sqrt[3]{y^3})^{11}$.

305. The third term of $\sqrt[3]{(a^2 + x)}$.

306. The third term of $\frac{1}{(a + b)^2}$.

307. The third term of $\frac{a}{(a - x)^{\frac{1}{2}}}$.

308. The eighth term of $\left(5 - \frac{z}{5}\right)^{10}$.

309. The ninth term of $\left(\frac{2}{x} - \frac{x}{4}\right)^{10}$.

310. The fifth term of $\left(a^{\frac{1}{2}} + \frac{x^2}{3}\right)^{-\frac{2}{3}}$.

311. The fifth term of $\left(3a - \frac{1}{\sqrt{2}}\right)^{-\frac{3}{2}}$.

312. The fifth term of $(a + \sqrt{2}x)^{-\frac{3}{2}}$.

313. The fifth term of $(6a^3 + \sqrt[3]{x^{-2}})^{-4}$.

314. The fifth term of $\left(a^{\frac{1}{2}} - \frac{x^{\frac{1}{2}}}{2a}\right)^{\frac{3}{2}}$.

315. The fifth term of $\left(\frac{a-x^{\frac{3}{2}}}{a^{\frac{1}{2}}-x^{\frac{1}{2}}}\right)^{-2}$.

316. The eighth term of $\left(2a^{\frac{1}{2}} + \frac{1}{2a}\right)^{11}$.

317. The fifth term of $\frac{xy^2}{\left(\frac{x}{2} + \frac{2y}{3}\right)^7}$.

318. The sixth term of $\left(\frac{a}{4} - \sqrt[n]{b^n}\right)^{30}$.

319. The fifth term of $(3a^2 - \frac{3}{4}b^2)^{12}$.

320. The fifth term of $(2 - 2\sqrt[4]{x^3})^{-\frac{3}{2}}$.

321. The fourth term of $(4 + \frac{2}{3}x^{2c})^{-\frac{5}{2}}$.

322. The fourth term of $ax^2(a^{-m} + x^{-\frac{2}{3}})^{-\frac{2}{3}}$.

323. The fourth term of $\frac{2ax}{(a^{-\frac{1}{2}} - x^{-\frac{2}{3}})^{-\frac{2}{3}}}$.

324. The sixth term of $\left(\frac{4}{3a} - x^{-\frac{3}{2}}\right)^{-3}$.

325. The sixth term of $3a\left(2 + \frac{1}{x}\right)^{-4}$.

326. The third term of $\left(\frac{1}{8} - \sqrt{x^3}\right)^{-\frac{3}{2}}$.

327. The third term of $\left(3\frac{3}{8}a + \frac{x}{4}\right)^{\frac{4}{3}}$.

328. The third term of $\sqrt[3]{a\left(\frac{4}{a} - x^{-2}\right)^{-\frac{7}{2}}}$.

USE OF LOGARITHMS.

Perform by logarithms the operations indicated :

329. $\frac{7340 \times 3549}{681.8 \times 593.1}$.

330. $\frac{654 \times 640 \times .3691}{87 \times 9 \times .045}$.

331. $\frac{.69 \times 7.5 \times 32.71 \times .003}{87 \times 8968 \times .0008}$.

332. $\left(\frac{21}{373}\right)^{\frac{2}{3}}$.

336. $\sqrt[9]{1350\frac{7}{8}}$.

337. $\sqrt[8]{172\frac{5}{6}}$.

333. $\left(\frac{112}{1727}\right)^{\frac{3}{5}}$.

338. $\sqrt[13]{\frac{3348}{569}}$.

334. $\sqrt[7]{\frac{1171}{345}}$.

339. $\frac{(\sqrt[5]{146298})^4}{(\sqrt[6]{988789})^5}$.

340. $\frac{(52072)^{13} \times \sqrt[3]{(0.000734)^9}}{(255608)^8}$.

341. $\sqrt[4]{\frac{132 \times (7.356)^9}{\sqrt[3]{(3.25)^5}}}$.

342. $\frac{\sqrt[7]{(466871)^6} \times \sqrt[9]{(3576)^{16}}}{996003 \sqrt[4]{0.0071}}$.

343. $\sqrt[3]{(0.26 \sqrt{\frac{2}{3}})}$.

344. $\sqrt{\frac{\sqrt[5]{3425} \sqrt[7]{136}}{0.00034}}$.

345. $253 \sqrt[3]{\frac{716.5}{\sqrt{2}}}$.

346. $\left(\frac{42666}{1147}\right)^{12} \times \left(\frac{765}{19432}\right)^{10}$

347. $\sqrt[5]{(\frac{7}{3} \sqrt[4]{6})}$.

348. $\frac{(\frac{2}{3})^{\frac{1}{2}} \times (\frac{3}{4})^{\frac{1}{3}}}{17\frac{1}{2}}$.

349. $\frac{1}{7} \sqrt[5]{\frac{5}{8}} \times .012 \sqrt[3]{\frac{7}{11}}$.

350. $\frac{\frac{1}{5} \sqrt[3]{\frac{11}{2}} \times .03 \sqrt[3]{15\frac{1}{5}}}{7\frac{1}{2} \sqrt[3]{12\frac{1}{5}} \times .19 \sqrt[4]{17\frac{1}{8}}}$.

351. $\frac{127}{4} \left(\frac{\sqrt[5]{19} + \frac{4}{7} \sqrt[3]{35\frac{1}{5}}}{14\frac{7}{9} - \frac{1}{11} \sqrt[5]{28\frac{2}{3}}} \right)$.

352. $\left(\frac{5}{7}\right)^{0.0587}$.

353. $(2\frac{5}{8})^9$.

354. $\left(\frac{643}{637}\right)^{123}$.

355. $\sqrt[3]{(21 + \sqrt[6]{19})}$.

356. $\sqrt[3]{(5.03 + \sqrt[5]{0.2})}$.

357. $\sqrt[5]{(9.921 - 3 \sqrt[3]{5.02})}$.

358. $\sqrt[16]{\frac{43 + 5 \sqrt[3]{278}}{\sqrt[5]{17}}}$.

359. $(317\frac{3}{4})^{0.6}$.

360. $\left(\frac{167}{53}\right)^{0.32}$.

361. $\frac{(991.767)^5 \times 12.34}{(20.358 \times 10.1575)^6}$.

In the following, find a fourth proportional to the three given numbers:

362. 37.125, 14.768, and 135.279.

363. .05764, .7186, and .34721.

364. 12.678, 14.065, and 100.979.

365. 1.9864, .4678, and 50.4567.

366. .09658, .24958, and .008967.

367. Find a third proportional to 12.796 and 3.24718.

Perform the following without tables:

368. Given $\log 2 = 0.3010300$ and $\log 3 = 0.4771213$; find logs of 4, 6, 8, 9, and 12.

369. Given $\log 2 = .3010300$; find $\log 128$, $\log 125$, and $\log 2500$.

370. Given $\log 2 = .3010300$ and $\log 7 = .8450980$; find the logs of 50, .005, and 196.

371. Given $\log 2 = .3010300$ and $\log 3 = .4771213$; find the logs of 6, 27, 54, and 576.

372. Given $\log 2 = .3010300$, $\log 3 = .4771213$, and $\log 7 = .8450980$; find logs 60, .03, 1.05, and .0000432.

373. Given $\log 2 = .3010300$, $\log 18 = 1.2552725$, and $\log 21 = 1.3222193$; find $\log .00075$, and $\log 31.5$.

374. Given $\log 25 = 1.3979400$ and $\log 1.03 = .0128372$; find the logs of 5, 4, 51.5, $(.064)^{\frac{1}{10}}$.

375. Given $\log 1.03$ as before, how many digits are there in the integral part of $(1.03)^{600}$?

376. Given $\log 4 = .6020600$ and $\log 1.04 = .0170333$; find the logs of 2, 25, 83.2, $(.625)^{\frac{1}{10}}$.

377. Given $\log 1.04$ as before, how many digits are there in the integral part of $(1.04)^{6000}$?

Solve the equations:

$$378. 4096^x = \frac{8}{64^x}.$$

$$379. a^x b^x = m.$$

$$380. \left(\frac{1}{.4}\right)^x = 6.25.$$

$$381. a^{mx} b^{2x} = c.$$

$$382. 2^{x+1} + 4^x = 80.$$

